5.8. Multi-Variable Optimization

Profit = Revenue – Costs

Revenue = Demand function * Quantity

Maximum and Minimums

A summary of the necessary and sufficient conditions for establishing local maxima and minima of functions of two variables is as follows:

Condition	Local maximum	Local minimum
First-order necessary	$f_x = f_y = 0$	
Second-order sufficient	$f_{xx}, f_{yy} < 0$	$f_{xx}, f_{yy} > 0$
	$f_{xx}f_{yy} - f_{xy}^2 > 0$	

Second order conditions

$|H| = f_{xx}f_{yy} - f_{xy}^2$. is the determinant of the quadratic d(dz)

Saddle Points

Saddle points exist if at the point (xo,yo) **satisfies the first order conditions but not the second order conditions.**

Constrained Optimisation - Substitution

Let's say we have a constraint (e.g. a budget line) that specifies some limit on x or y. For example:

Using the function

$$z = f(x, y) = 2 - (x - 1)^2 - (y - 1)^2 \ ,$$

and the constraint $2 = y + \frac{2}{5}x$, find the new optimum point.

We can use the substitution method to solve this. However this only works when:

1. The constraint is relatively simple
2. The constraint can therefore be rearranged to perform substitution into the objective function.

When this is not the case, we need to use the **Lagrange Multiplier Method**

Constrained Optimisation – Lagrange Multiplier Method

The lagrange multiplier helps when the constraints are more difficult, such as -6 = x + y + z.2

$$F(x, y, z, \lambda) = f(x, y, z) - \lambda g(x, y, z)$$

Where f(x,y,z) is our normal function and g(x,y,z) is our constraint function. You express the constraint function as everything = 0. For example:

$$g(x, y, z) = x - y + 2z - 6 = 0.$$