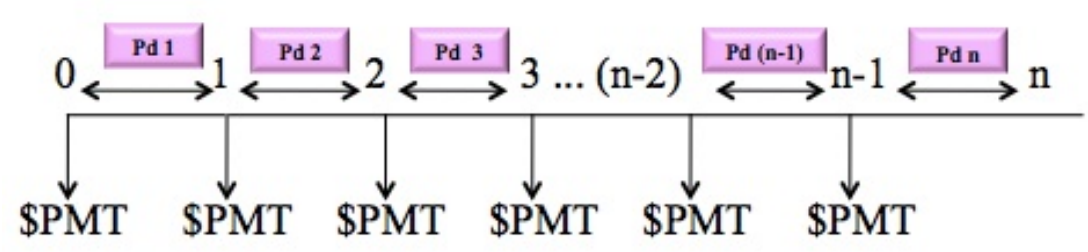
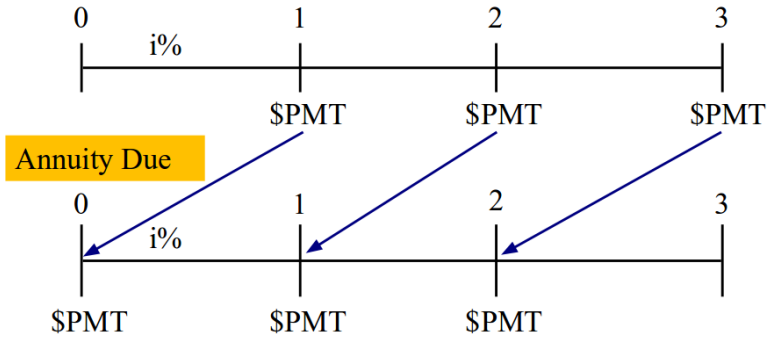
Annuities due:
If the cash payments are made or received at the **beginning (start)** of each period then the annuity is referred to as an **annuity due** or annuities in advance.



Ordinary Annuity



Annuity Due

Future and Present Values of Ordinary Annuities

FV Calculation:

Where:
FV = the accumulated or future value of the annuity
PMT = the cash flow received/paid under the annuity
n = the time period over which the annuity occurs
i = the per-period interest rate

$$FV = \frac{PMT}{i}\left[(1+i)^n - 1\right]$$

OR FV = PMT X $(1+i)^n$ – 1/ i

$$\text{Future value of an annuity} = PMT\left[\frac{\text{future value factor} - 1}{r}\right]$$

$$= PMT\left[\frac{(1 + r)^n - 1}{r}\right]$$

E.g. at the end of every year for 5 years, you put $500 in the bank to save towards purchasing a car. Goal at the end of 5 years, you earn interest or some type of return e.g. 7%. What is the value at the end of that period?

PMT = $500
INTEREST = 7% = 0.07
N = 5

= 500 x $(1+0.07)^5$ – 1/ 0.07
= $2875.37

Therefore, if you were to deposit $500 at the end of every year for 5 years, with an interest rate of 7%, you will earn $2875.37 at the end of year 5.

PV Calculation:

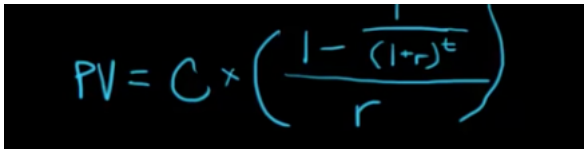
$$PV = \frac{PMT}{i}\left[1 - \frac{1}{(1+i)^n}\right]$$

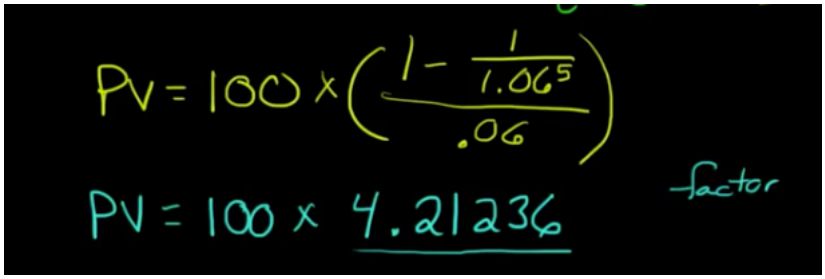
$$\text{Future value of an annuity} = PMT\left[\frac{\text{future value factor} - 1}{r}\right]$$

$$= PMT\left[\frac{(1 + r)^n - 1}{r}\right]$$

E.g. At the end of every year for 5 years, we have a stream of $100 payments.

Interest = 6% = 0.06
N = 5
PMT = 100





=$421.40 (Not $500, because of the time value of money)

E.g. 2 The annual cash flows a corporation receives from an asset they have invested in are $2.3 million paid at the end of each year from year 1 to year 6. The interest rate they receive is 10% p.a. compounded annually.

- What is the accumulated future value of this asset at the end of year six?
- What is the present value of this asset?

FV of Annuity Due = FV of Ordinary Annuity × (1+i)

$$FV = \frac{PMT}{i}\left[(1+i)^n - 1\right] \times (1+i)$$

PV of Annuity Due = PV of Ordinary Annuity × (1+i)

$$PV = \frac{PMT}{i}\left[1 - \frac{1}{(1+i)^n}\right] \times (1+i)$$

Example:

Kathy's rich uncle promises her $1,000 **per month**, starting **today**, with a final payment to be made 6 months from today. If the interest rate is 6 percent **per annum**, what is the present value of the cash flows?

Important note: look at the payment frequency - here it is **monthly**!

The rate given however is **annual.** This needs careful consideration. What needs to be done? Convert the annual interest rate to a monthly one, that is, 6% divided by 12 is equal to a monthly interest of 0.5%. **Note** also that as there is a final payment at time 6, the timeline is 7 years long with the payment at time 6 being at the START of the last period (by definition, an annuity due has payments made at the START of each period).