STIGLITZ PARADOX

If:

1. Markets are efficient and securities' prices reflect all available information and,
2. Obtaining information about securities requires resources

Then:

1. Why do people commit resources to research securities at all, and
2. If people don't need to commit resources to researching securities, then how did the prices get right to begin with?
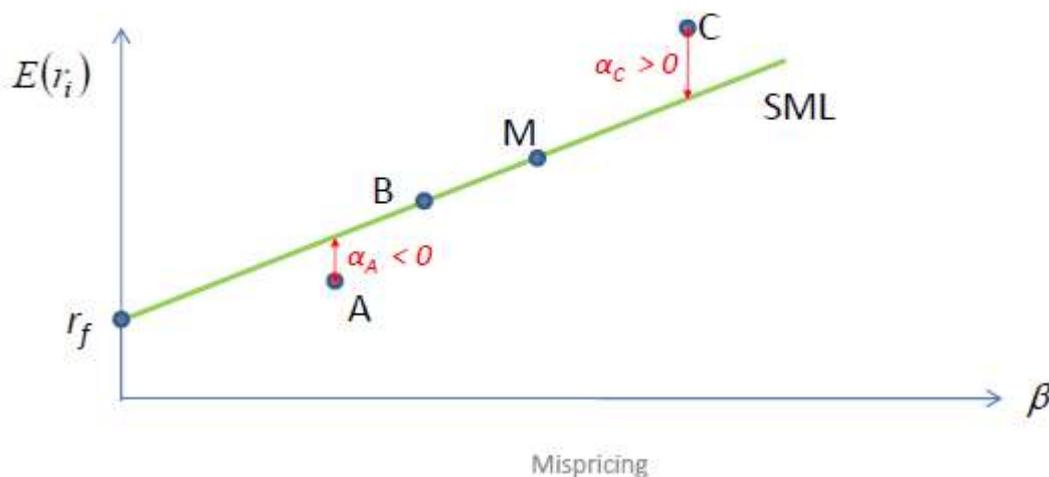
According to CAPM, everyone should buy the market portfolio, so nobody has any incentive to do the analysis.

SINGLE INDEX MODEL

The single index model may be expressed by the following equation, derived from CAPM:

$$r_{it} - r_{ft} = \alpha_i + \beta_i \left[r_{Mt} - r_{ft} \right] + \varepsilon_{it}$$

M being the market index. If the Alpha =/= 0, the asset is mispriced. If an asset is correctly priced, it should be plotted along the SML.

Mispricing

EXPLOITING MISPRICING

If we identify assets are mispriced, we want to deviate from the market weights to exploit the mispricing. This means selecting an optimal risky portfolio, P*, that is not equal to the market portfolio. By doing so we can dominate the market portfolio.

To do this, we combine some mispriced asset with the market portfolio to form the new optimal risky portfolio with the highest Sharpe ratio.

$$\max_{w_A} S_P = \frac{E[r_P(w_A)] - r_f}{\sigma_P(w_A)}, \text{ where } \quad r_P = w_A r_A + (1 - w_A) r_M$$

When we take an active position, we exploit alpha but incur some unsystematic risk.

$$r_{At} - r_{ft} = \alpha_A + \beta_A [r_{Mt} - r_{ft}] + \varepsilon_{At}$$

When solving the maximization problem, we weigh these effects against each other. We can think as if we were buying the benefit of mispricing at the cost of taking on unsystematic risk.

The price at which we buy the mispricing is:

$$\alpha_A / \sigma^2_{\varepsilon A}$$

This is incorporated in finding the optimal weight of a mispriced asset:

$$\sigma^2_A = \beta^2 \sigma^2_M + \sigma^2_\varepsilon \Leftrightarrow \sigma^2_\varepsilon = \sigma^2_A - \beta^2 \sigma^2_M$$

HAVE TO INCLUDE SQUARE SIGNS IF DEALING WITH STANDARD DEVIATION OR FORMULA DOESN'T WORK FOR SOME REASON

$$w^*_A = \frac{w^0_A}{1 + w^0_A(1 - \beta_A)}$$

$$\text{where, } w^0_A = \frac{\alpha_A}{\sigma^2_{\varepsilon A}} \bigg/ \frac{E(r_M) - r_f}{\sigma^2_M}$$

The result of w* tells you how much to invest in the risky investment.

INFORMATION RATIO, EFFECTS OF MISPRICING ON SHARPE RATIO

The information ratio is:

$$IR = \frac{\alpha}{\sigma_\varepsilon}$$

It is used to evaluate individual assets in an active portfolio especially when creating active portfolio of mispriced assets with the market portfolio. The higher the absolute value of the information ratio, the higher its contribution to the Sharpe ratio of the final optimal risky portfolio.

$$S_P^2 = S_M^2 + \left(\frac{\alpha}{\sigma_\varepsilon}\right)^2$$

The Ir^2 of the active portfolio will equal the sum of squared information ratio of individual active eassets.

$$- IR^2{}_{AP} = \sum_j IR^2{}_j$$

FAMA-FRENCH-CARHART FOUR-FACTOR MODEL

$$E(r_i) = r_f + \beta_i^M \left[E(r_M) - r_f\right] + \beta_i^{SMB} E(r_{SMB}) + \beta_i^{HML} E(r_{HML}) + \beta_i^{MOM} E(r_{MOM})$$

Each new factor is *self-financing* in that it takes long (buy) and short (sell) positions that offset each other so that the net portfolio cost is zero. You cannot assume anything about an individual stock based on factor loadings – because it measure the covariance of Microsoft's return with a particular strategy.

To conclude anything about the optimal portfolio, there must be some info about risk.

- The Small-mius-Big portfolio, SMB, consists of a long position in the market's 50% smallest firms by market cap (*Small*) and a short position in the market's 50% largest firms (big)
- The High-minus-low portfolio *HML*¸ consists of a long position in the 50% of the market with the highest Book-to-Market ratios (*value stocks*) and a short position in the 50% of the market with the lowest Book-to-market ratios (*Growth stocks)*. Essentially, you buy top 50% sell bottom 50%. A +-tive book-to-market implies that high book-to-market stocks outperformed low ones.
- The Momentum Portfolio, *MOM*, takes a long position in the 30% of the market that had the highest returns the preceding year (*winners*) and a short position in the 30% of the market that had the lowest returns the preceding year (*losers*)
- The loading of any asset to each factor depends on its covariance with the factor