T-TESTS FOR A SINGLE MEAN I

<u>Use this test when σ_x^2 is not known & must be estimated to find the probability of getting an outcome beyond a certain value</u>

Outcome could be:

- Critical *t*
- Critical $\underline{\bar{X}}$
- Observed *p*

BIASED & UNBIASED ESTIMATORS

The sample variance is a biased estimator of the population variance – if take all possible samples of size *n* from a given population & calculated the variance for each of those samples & then calculated the mean of the sample variances would get a value that is less than the population variance

$$mean\ of\ sample\ variances\ \neq true\ variance\ of\ population$$

Need to correct for this bias by increasing the standard deviation

$$Sample\ Variance = \frac{population\ variance}{n} = \frac{\hat{\sigma}_x^2}{n}$$

$$Population\ Variance = \ \hat{\sigma}_x^2 = \frac{\sum(X - \bar{X})^2}{n - 1}$$

Where:

- $(X - \bar{X})^2$ = sum of squares (SS)
- $n - 1$ = degrees of freedom (df)

N.B. May also see $\hat{\sigma}_x^2$ written as s^2x

DEGREES OF FREEDOM

Degrees of Freedom (df): Number of observations minus the number of things being estimated

n observations being used to estimate 1 mean:

$$df = n - 1$$

Df are the number of independent pieces of information needed to determine a statistic

t – Test for a Single Mean

T-formula:

$$t = \frac{\bar{X} - \mu}{\sigma_{\bar{X}}}$$

$$\hat{\sigma}_{\bar{X}} = \frac{\hat{\sigma}_{\bar{X}}}{\sqrt{n}}$$

$$\hat{\sigma}_{\bar{X}} = \sqrt{\frac{SS}{df}}$$

T-TEST VS. Z-TEST

If population standard deviation is known, then do a z-test:

$$\sigma_x \rightarrow (divide\ by\ \sqrt{n})\ \sigma_{\bar{X}}$$

If population standard deviation *not known* (but estimated), then do a t-test:

$$\hat{\sigma}_x \rightarrow (divide\ by\ \sqrt{n}\ )\ \hat{\sigma}_{\bar{X}}$$

Estimated population standard deviation:

$$S_x \rightarrow (divide\ by\ \sqrt{n}\ )\ S_{\bar{X}}$$

$$raw\ scores \rightarrow sample\ means$$

(this is the same as $\hat{\sigma}_x \rightarrow \hat{\sigma}_{\bar{X}}$)

Ways To Test Hypotheses With t-Tests For A Single Mean

Determine the research question

Formulate the statistical hypotheses (i.e. null hypothesis H_0 & alternative hypothesis H_A)

- Directional (one-tailed) vs. non-directional (two-tailed)

Three ways to test statistical hypotheses:

1) Critical t: Find critical t-score based on α & then calculate and compare observed t
2) Critical $\bar{X}$: Find critical t-score, substitute into t formula to find critical $\bar{X}$ & compare with observed $\bar{X}$
3) Observed P: Calculate observed t & find corresponding *p* value & compare to α

N.B. It is unrealistic that the population variance would ever be known, hence usually use one-sample t-test

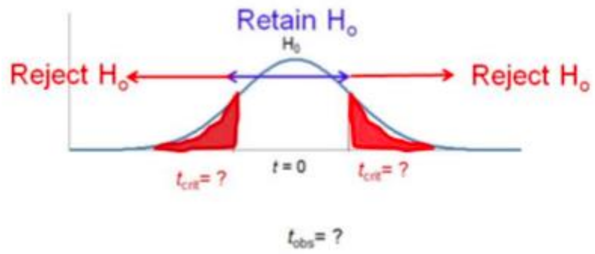
CRITICAL t METHOD

Find t_{CRIT} based on α calculate & compare t_{obs}

EXAMPLE

Research question: Does exercise influence wellbeing for people with depression?
Variables:

- $\mu_x = 20$
- $\sigma_x = ?$
- $n_x = 10$
- $\bar{X} = ?$
- $\alpha = 0.05$



Hypotheses:

- H$_0$: $\mu_x = 20$
- H$_A$: $\mu_x \neq 20$

These are non-directional hypotheses, hence use a two-tailed test

$\hat{\sigma}_x = 3.80$, $\hat{\sigma}_{\bar{X}} = 1.20$, $t_{obs} = \frac{\bar{X}-\mu}{\hat{\sigma}_{\bar{X}}} = 1.66$, $df = 9$

Using t-table – unlike z, each value of t does not have its own p value, p depends on value of t & the df

$$t_{crit}(9) = \pm 2.26$$

Observed t (1.66) < critical t hence retain H$_0$ since is not in the rejection region

The mean psychological wellbeing scores of a group of people with depression who completed the exercise regimen was not statistically significantly different from the usual mean, t(9)=1.66, *ns*.

N.B. Use language such as there was no significant, there was no evidence of, there did not appear to be etc. as have only made statistical inferences (have not proved anything)

N.B. T- distribution is more peaked in the middle

CRITICAL $\bar{X}$ METHOD

Find t_{crit} based on α, substitute t_{crit} into t formula to find $\bar{X}_{crit}$, compare $\bar{X}_{crit}$ with $\bar{X}_{obs}$

EXAMPLE

Research Question: Does exercise influence wellbeing for people with depression

Variables:

- $\mu_x = 20$
- $\sigma_x = ?$

- $n_x = 10$
- $\bar{X} = 22$
- $\alpha = 0.05$
- $\hat{\sigma}_x = 1.2$

Hypotheses:

- H_0: $\mu_x = 20$
- H_A: $\mu_x \neq 20$

$$t_{crit} \times \hat{\sigma}_{\bar{X}} + \mu = \bar{X}_{crit}$$

$$(+2.26 \times 1.20) + 20 = \bar{X}_{crit-upper} = 22.71$$

$$(-2.26 \times 1.20) + 20 = 17.29$$

$$\therefore \bar{X}_{obs} > 17.29 \ \& < 22.71, retain\ H_0$$

OBSERVED *p* METHOD

Calculate t_{obs}, find corresponding p-value, compare with α

This requires a stats package – hence would rarely use this method