VALUING BONDS

Bond: Security that obligates the issuer to make specified payments to the bondholder

- ⇒ Long-term debt instruments issued by governments & corporations for the purpose of borrowing money/raising funds
- ⇒ Owners of a bond receive interest payments over the life of the bond & at maturity also get back the principal amount

Face Value: (par value or principal value) Payment at the maturity of the bond, nominal amount of principal borrowed, amount used to compute interest payments over the life of the bond
Coupon: The interest payments made to the bondholder
Coupon Rate: Annual interest payment expressed as a percentage of face value, set by the issuer & stated on the bond certificate, expressed as an APR
N.B. The coupon rate is not the discount rate used in the present value calculations

- ⇒ The coupon rates merely tells us what cash flow the bond will produce

Coupon Payments: Periodic interest payments determined by coupon rate, amount of each coupon payment,

$$CPN = \frac{Coupon\ Rate \times Face\ Value}{Number\ of\ Coupon\ Payments\ per\ Year}$$

Maturity Date: Final repayment date of bond (inc. principal amount borrowed)
Bond Certificate: States the terms of the bond, amounts & dates of all payments to be made
Term: Time remaining until payment due

Valuing A Bond

The price of a bond is the present value of all cash flows generated by the bond (i.e. coupons & face value) discounted at the required rate of return

$$PV = \frac{cpn}{(1+r)^1} + \frac{cpn}{(1+r)^2} + \cdots + \frac{(cpn + par)}{(1+r)^1}$$

EXAMPLE
In October 2014 you purchase 100 euros of bonds in France which pay a 4.25% coupon every year, if the bond matures in 2014 & the yield to maturity (YTM) is 0.15%, what is the value of the bond?

$$PV = \frac{4.25}{1.0015} + \frac{4.25}{(1.0015)^2} + \frac{4.25}{(1.0015)^3} + \frac{104.25}{(1.0015)^4} = 116.34$$

Value of the bond = total PV of all coupon payments & face value of bond
Determine desired yield (required rate of return) – look at return provided from similar securities

SHORT-CUT
PV (bond) = PV (coupon payments) + PV (principal)
= (coupon payment x n-year annuity factor) + (face value x n-year discount factor)

$$PV = cF\left[\frac{1}{r} - \frac{1}{r(1+r)^n}\right] + \frac{F}{(1+r)^n}$$

Where:

- $C = cf$
- F= face value

N.B. Coupon and yield rate must have matching compounding frequency

YIELD TO MATURITY (YTM) OF BOND

Internal rate of return (IRR) on an interest bearing instrument
YTM summarizes its prospective return given its observed market price
To calculate the YTM on the n-year, need to solve for r in the following equation:

$$Bond\ Price = \frac{C}{(1+r)} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3} + \cdots + \frac{F+C}{(1+r)^n}$$

YTM discounts all cash payments at the same rate (even if spot rates differ)
N.B. YTM can't be calculated until know the bond's price or PV

Price of a bond is the present value of all cash flows generated by the bond (i.e. coupons & face value) discounted at the required rate of return

$$PV = \frac{cpn}{(1+r)^1} + \frac{cpn}{(1+r)^2} + \cdots + \frac{(cpn + par)}{(1+r)^t}$$

EXAMPLE

If today is October 1 2015, what is the value of the following bond? An IBM Bond pays $115 every September 30 for 5 years, in September 2020 it pays an additional $1000 & retires the bond, bond is rated as AAA (YTM for AAA is 7.5%)

$$PV = \frac{115}{1.075} + \frac{115}{(1.075)^2} + \frac{115}{(1.075)^3} + \frac{115}{(1.075)^4} + \frac{1115}{(1.075)^5}$$

$$= \$1161.84$$

Where:

- F=1000
- C=115
- R=0.075
- n=5

If not given in question, assume a government bond has a face value of $1000

VALUING A BOND AS AN ANNUITY

PV (bond) = PV (annuity of coupons) + PV (principal)

$$PV = (cpn \times PVAF) + (final\ payment \times discount\ factor)$$

$$= 4.25 \times \left[\frac{1}{0.0015} - \frac{1}{0.0015(1+0.0015)^4}\right] + \frac{100}{(1+0.0015)^4}$$

$$= 116.34$$

N.B. To calculate bond price at a future date, change n to match how many periods remain

- ⇒ E.g. For a five year bond, if asked to find price one year from now, change time period (n) to be 4 instead of 5 (because there are 4 periods left after 1 year)

Return received on a bond between two time periods: (e.g. period 0 (today) & period 1 (one year from now))

$$Rate\ of\ Return = \frac{PV_1 - PV_0 + C}{PV_0}$$

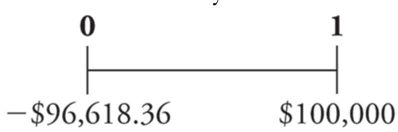
ZERO-COUPON BONDS

Bonds that pay a single fixed amount (face value) at a fixed date in the future (maturity) issue no coupon payments

Only two cash flows – bond's market price at time of purchase & face value at maturity

Always sells at a discount (price lower than face value) – hence also called pure discount bonds

Suppose that a one-year, risk-free, zero-coupon bond with a $100 000 face value has an initial price of $96618.36, if purchased the bond & held it to maturity would have the following cash flows:



Although the bond pays no 'interest' your compensation is the difference between the initial price & the face value

YIELD TO MATURITY

Discount rate that sets the present value of the promised bond payments equal to the current market price of the bond

$$P = \frac{FV}{(1 + YTM_n)^n}$$

$$YTM_n = \left(\frac{FV}{P}\right)^{\frac{1}{n}} - 1$$

Dynamic Behavior of Bond Prices

Discount (P<F) - bond is selling at a discount if the price is less than the face value of the bond
Par (P=F) – bond is selling at par if the price is equal to the face value of the bond
Premium (P>F) – bond is selling at a premium if the price is greater than the face value of the bond

DISCOUNTS & PREMIUMS

- If a coupon bond sells at par (P=F), the only return investors will earn is from the coupons that the bond pays
- ⇒ Bonds YTM will be exactly equal to its coupon rate (c=r)
- If a coupon bond trades at a discount (P<F), an investor will earn a return both from receiving the coupons and from receiving a face value that exceeds the price paid for the bond
- ⇒ Bond's YTM will exceed its coupon rate (c<r)
- If a coupon bond trades at a premium (P>F) investors will earn a return from receiving the coupons but this return will be diminished by receiving a face value less than the price paid for the bond
- ⇒ Bond's YTM will be less than its coupon rate (c>r)

N.B. If c>r, investors will expect a decline in the capital value of the bond over its remaining life, hence bond price will be greater than face value

$P = C\left[\frac{(1+r)^n - 1}{r(1+r)^n}\right] + \frac{F}{(1+r)^n}$ & using C=cF (c=coupon rate)

$$\rightarrow P = cF\left[\frac{(1+r)^n - 1}{r(1+r)^n}\right] + \frac{F}{(1+r)^n}$$

Subtracting F from both sides & other algebra:

$$P - F = F[c - r]\left[\frac{(1+r)^n - 1}{r(1+r)^n}\right]$$

$$\therefore sign[P - F] = sign[c - r]$$

If:

$$c > r \Rightarrow P > F$$
$$c < r \Rightarrow P < F$$
$$c = r \Rightarrow P = F$$

Zero-coupon bonds always trade for a discount
Coupon bonds may trade at a discount or at a premium to par value
Most issuers of coupon bonds choose a coupon rate so that the bonds will initially trade at, or very close to, par
After the issue date, the market price of a bond changes over time

When the bond price is . . .	greater than the face value	equal to the face value	less than the face value
We say the bond trades . . .	"above par" or "at a premium"	"at par"	"below par" or "at a discount"
This occurs when . . .	Coupon Rate > Yield to Maturity	Coupon Rate = Yield to Maturity	Coupon Rate < Yield to Maturity

Bond Prices & Yields

Inverse relationship exists between bond prices & bond yields (interest rates)

$$\uparrow bond\ yield\ (r) \Rightarrow \downarrow bond\ price$$
$$\downarrow bond\ yield \Rightarrow \uparrow bond\ price$$
$$\uparrow bond\ maturity \Rightarrow \uparrow sensitivity\ of\ price\ to\ YTM$$

Bond curve is relatively linear – short maturity period on average
If there is a higher interest rate (yield), will discount the bond at a higher rate, this will reduce the present value & hence price