Foundations of Physics

PHYS130: Comprehensive Notes

SESSION 2, 2017
MACQUARIE UNIVERSITY

BLUE TEXT – Titles and subheadings

BLACK (BOLDED & ITALICISED) – Key terms

Underlined – Essential concepts that require emphasis

Green Text – Important equations or relationships

RED TEXT – Example questions and worked answers

Table of Contents

Week 03 – Force & Motion

FORCES

A ***FORCE*** ($\vec{F}$) is a push or pull on an object. It acts on an object to change its velocity, thus causing acceleration. If forces on a body *balance* one another, then system is in ***EQUILIBRIUM*** and the vector sum of forces is zero. Important features of force include:

- Force is a vector quantity, with both magnitude and direction. If two or more forces act on a body, the ***NET FORCE*** ($\vec{F}_{net}$) is found by adding the individual vector forces. The net force points in the direction of the resulting change in velocity.

- The unit of force is defined in terms of the acceleration a force would give to the standard kilogram. If an object with mass $1\ kg$ is placed on a frictionless surface and pulled horizontally, the object will have an acceleration of $1\ m/s^2$. The applied force thus has a magnitude of 1 ***NEWTON*** (N).

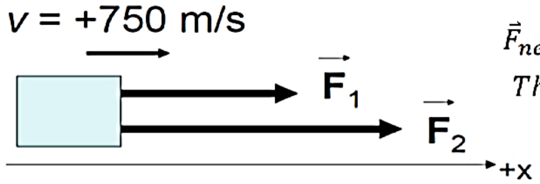
$$1N = 1kgm/s^2$$

DRAWING DIAGRAMS

Often several forces act on a body in different directions. A ***FREE-BODY DIAGRAM*** is always first constructed, with each force drawn as a vector with its tail anchored on the body.

A ***SYSTEM*** consists of one or more bodies, and any force on the bodies *inside* the system from bodies *outside* the system is called an ***EXTERNAL FORCE.*** If the bodies making up the system are rigidly connected to one another, we can treat the system as one composite body, and the net force on it is the vector sum of all external forces. ***INTERNAL FORCES***—forces between bodies inside the system—cannot accelerate the system.

EXAMPLE 1:
The figure below shows two forces with magnitudes F_1 = +2000 N and F_2 = +3000 N acting on an object, the plus signs indicating that the forces act along the +x axis. A third force $\vec{F}_3$ also acts on the object but is not shown in the figure. The object is moving with a constant velocity of +750 m/s along the x axis. Find the magnitude and direction of $\vec{F}_3$.



$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = +2000 + 3000 + \vec{F}_3 = 0$$

$$Thus\ \vec{F}_3 = -5000N$$

NEWTON'S FIRST LAW

NEWTON'S FIRST LAW: In the absence of an applied net force, a body cannot accelerate and will either remain at rest or continue moving with the same constant velocity (in the same direction with the same magnitude). This is because all objects have ***INERTIA*** – the tendency to resist motion unless a force acts upon it. If a body's velocity is constant, it can be assumed that the net force is zero.

NEWTON'S SECOND LAW

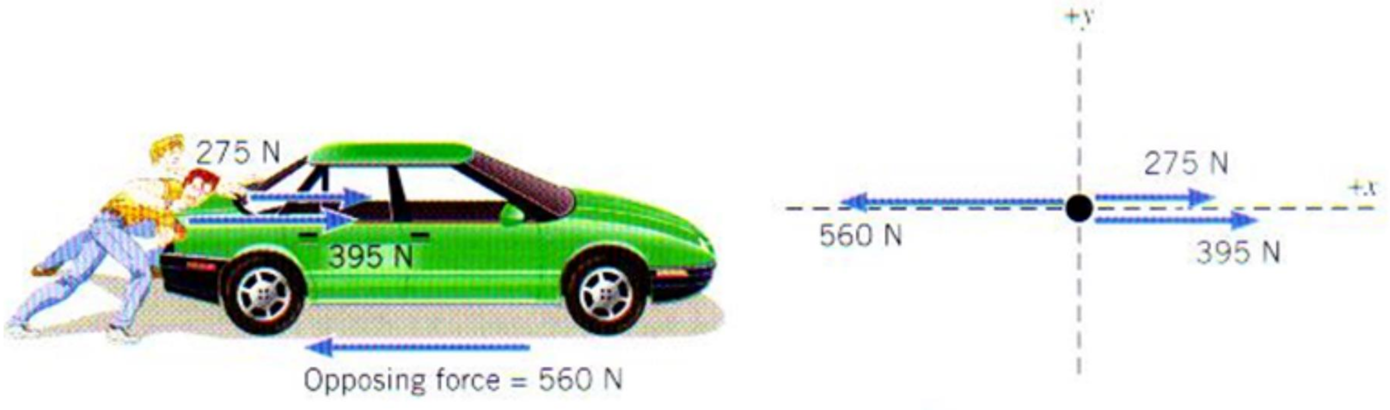
NEWTON'S SECOND LAW: The net force on a body is equal to the product of the body's mass and its acceleration. The direction of the acceleration is in the direction of the applied net force.

$$\vec{F}_{net} = m\vec{a} \qquad [1]$$

The acceleration component along a given axis is caused only by the sum of the force components along that *same* axis, and not by force components along any other axis.

$$\vec{F}_{net,x} = m\vec{a}_x \quad \vec{F}_{net,y} = m\vec{a}_y \quad \vec{F}_{net,z} = m\vec{a}_z$$

EXAMPLE 2: Two people push a stalled car, which has a mass of $1850kg$. Person A applies a force of $275N$ to the car, and the other applies a force of $395N$. Both forces act in the same direction, horizontally. A third force of $560N$ acts on the car due to friction, opposite to the direction in which the people are pushing. Find the acceleration of the car.



(a) (b) Free-body diagram of the car

The net force is: $\sum F = +275N + 395N - 560N = +110N$. Thus $a = \frac{\sum F}{m} = \frac{+110N}{1850kg} = +0.059m/s^2$.

FORCES OF IMPORTANCE

1. ***WEIGHT*** ($\vec{W}$): is a force due to gravity. Hence, weight and mass have distinct meanings. Weight depends on location, but mass does not. $\boldsymbol{W = mg}$

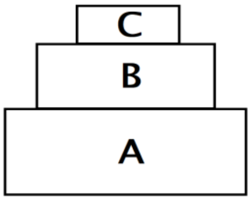
2. ***GRAVITATIONAL FORCE*** ($\vec{F}_g$): is the pull on an object that is directed toward a second body (most commonly the earth's centre). $\boldsymbol{F_g = -mg}$

3. ***NORMAL FORCE*** ($\vec{F}_N$): When a body presses against a surface, the surface deforms and pushes back on the body with a normal force that is perpendicular to the surface. Normal force is the force exerted by a surface on an object, perpendicular to the surface. $\boldsymbol{F_N = |F_g| = mg}$

EXAMPLE 3: A 100kg block of wood is travelling with a constant velocity on ice. What is its normal force?

$\vec{F}_N = mg = 100 \times 10 = 1000N$

EXAMPLE 4: Box A has mass 100kg, box B has mass 50kg and box C has mass 25kg. What is the normal force exerted by the ground experience by box A? Box B?



$$\vec{F}_{N,A} = mg = 175 \times 10 = 1750N$$
$$\vec{F}_{N,B} = mg = 75 \times 10 = 750N$$

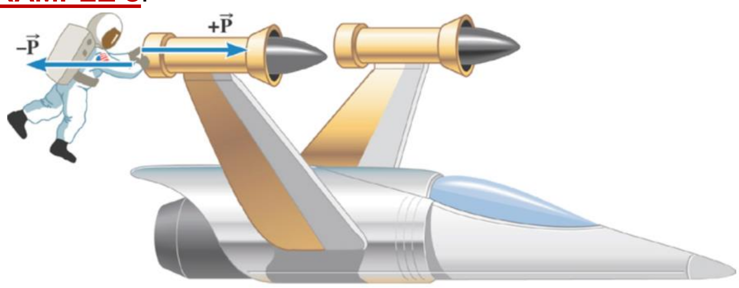
4. ***FRICTION*** ($\vec{f}$): is the force resisting the relative motion of surfaces sliding against each other. This force is directed along the surface, opposite the direction of the intended motion.

5. ***TENSION*** ($\vec{T}$): When a cord is attached to a body and pulled taut, the cord pulls on the body with a force $\vec{T}$ directed away from the body and along the cord. The cord is said to be *massless* and *non-stretchable* – it exists only as a connection between two bodies. It pulls on both bodies with the same force $\vec{T}$, even if the bodies and the cord are accelerating and even if the cord runs around a pulley. Tension is equal along the rope.

NEWTON'S THIRD LAW

NEWTON'S THIRD LAW: For every action there is an opposite and equal reaction. Two bodies are said to ***INTERACT*** when the forces exerted on each body from the other are equal in magnitude and opposite in direction. When any two bodies physically interact in any situation, a third-law force pair is present. For example, suppose you position a book *B* so it leans against a crate *C*. There is a horizontal force on the book from the crate and vice-versa.

EXAMPLE 5:



An astronaut pushes on the spacecraft with some force $\vec{\mathbf{P}}$. Suppose that the magnitude of the force is 36 N. If the mass of the spacecraft is 11,000 kg and the mass of the astronaut is 92 kg, what are the accelerations?

On the spacecraft $\sum \vec{\mathbf{F}} = \vec{\mathbf{P}}$.

On the astronaut $\sum \vec{\mathbf{F}} = -\vec{\mathbf{P}}$.

$$\vec{\mathbf{a}}_s = \frac{\vec{\mathbf{P}}}{m_s} = \frac{+36\text{ N}}{11{,}000\text{ kg}} = +0.0033\text{ m/s}^2$$

$$\vec{\mathbf{a}}_A = \frac{-\vec{\mathbf{P}}}{m_A} = \frac{-36\text{ N}}{92\text{ kg}} = -0.39\text{ m/s}^2$$

TYPES OF FRICTION

There are two types of frictional forces that oppose motion. Both act oppositely to the direction of motion that would occur in the absence of friction:

1. ***STATIC FRICTION*** (f_s) refers to the frictional force that keeps an object at rest. Static friction exists when there is no motion occurring along the contact surface. The maximum possible value of static friction is proportional to the normal force, and must be overcome to start moving an object. Given μ_s is the ***COEFFICIENT OF STATIC FRICTION***:

$$\boldsymbol{f_s \le \mu_s|\vec{F}_n| \quad and \quad f_s^{max} = \mu_s|\vec{F}_n|}$$

2. ***KINETIC FRICTION*** (f_k) is the frictional force that acts between moving surfaces. Its value is *constant* and proportional to the normal force. Given μ_s is the ***COEFFICIENT OF KINETIC FRICTION***:

$$f_k = \mu_k|\vec{F}_n|$$

The coefficient of friction depends on the *objects* that are causing friction. The value is usually between **0 and 1**. A value of:

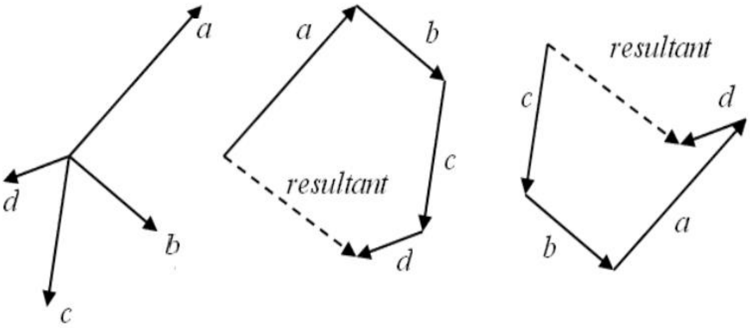
- $\mu = 0$ means there is no friction between the objects. This is only theoretically possible.
- $\mu = 1$ means the frictional force is equal to the normal force.

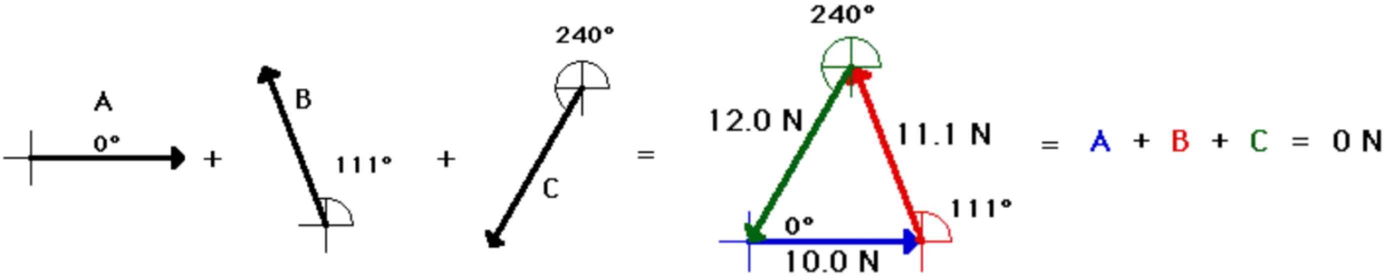
Week 04 – Forces in Two Dimensions

COMBINING FORCES GEOMETRICALLY

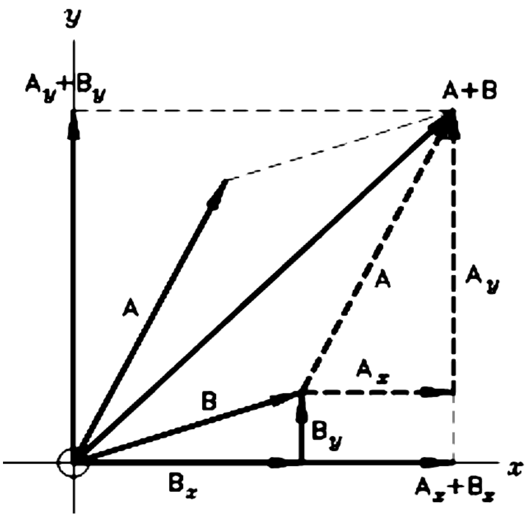
HEAD-TO-TAIL ADDITION can be used to find the vector sum of forces ($\vec{F}_{net}$). This requires an *accurately scaled* diagram.

The resultant vector can be determined by drawing a vector from the tail of the first vector to the head of the last vector. The order of vector addition is irrelevant, as the resultant vector is the same.





COMBINING FORCES BY COMPONENTS



The ***COMPONENTS*** of the resultant vector $\vec{R} = \vec{A} + \vec{B}$ are:

$$R_x = A_x + B_x \qquad \& \qquad R_y = A_y + B_y$$

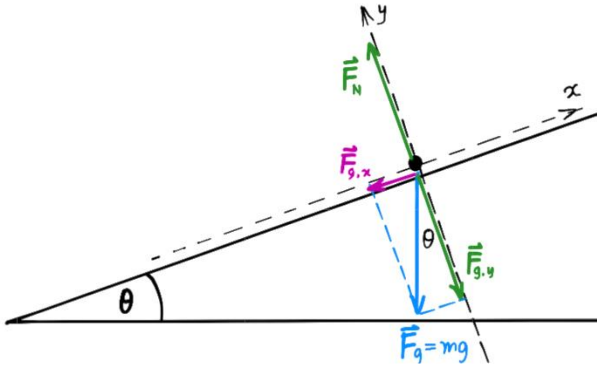
The ***LENGTH*** of the resultant vector is given by:

$$|\vec{R}| = \sqrt{R_x^2 + R_y^2}$$

The ***ANGLE*** that the resultant vector makes with the positive x-axis is given by:

$$\tan\theta = \frac{R_y}{R_x} \rightarrow \theta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$$

<u>EXAMPLE 1</u>: A car travelling at 30m/s runs out of petrol while travelling up a 5° slope. How far will it coast before starting to roll back down?



$First\ calculate\ the\ components\ of\ \vec{F}_g$:

$\vec{F}_{g,x} = F_g \sin\theta = mg\sin\theta\ \&\ \vec{F}_{g,y} = F_g\cos\theta = mg\cos\theta$

$In\ absence\ of\ petrol: \vec{F}_{net,x} = \vec{F}_{g,x}$

$Thus\ \vec{F}_{net,x} = -mg\sin\theta \qquad given\ that\ \vec{F}_{net} = ma$

$a = -g\sin\theta$

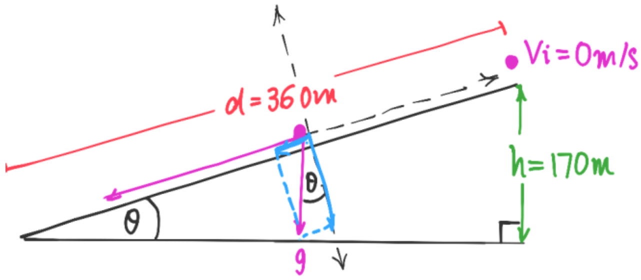
$To\ find\ the\ distance\ coasted, note\ that\ v_{f,x} = 0\ \&\ v_{i,x} = 30m/s$

$$v_{f,x}^2 = v_{i,x}^2 + 2a\vec{x} \rightarrow |\vec{x}| = \left|\frac{v_{f,x}^2 - v_{i,x}^2}{2a}\right| = \left|\frac{0 - 30^2}{2(-g\sin\theta)}\right| = \frac{30^2}{2\times 9.8\times\sin 5°} = 526m$$

EXAMPLE 2: The skier starts from rest and accelerates down a stretch of mountain with a constant slope. The distance travelled in this phase is 360m, and the vertical distance is 170m. What is the fastest speed the skier could achieve? What is the time taken to complete this run?

$Given\ v_i = 0, d = 360m\ and\ h = 170m$

$Note\ \sin(\theta) = \dfrac{170}{360} \rightarrow Thus\ \theta = \sin^{-1}\dfrac{17}{36} = 28°$



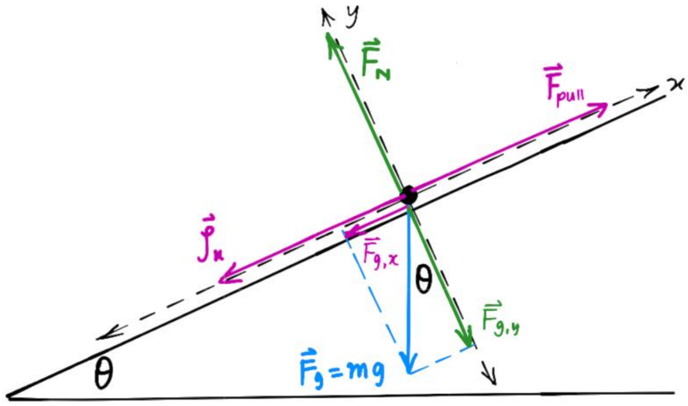
To find the acceleration along the slope:

$a = g\cos(90-\theta) = g\sin\theta = 9.8\sin 28°$

$v_f^2 = v_i^2 + 2ad = 0 + 2(9.8\sin 28°)(360) \approx 58m/s$

$t = \dfrac{v_f - v_i}{a} = \dfrac{58\ m/s}{4.6\ m/s^2} = 13s$

EXAMPLE 3: A father is pulling his child uphill on a sled in slushy snow. The child has a mass of 22kg, and the sled has a mass of 3.20kg. The coefficient of kinetic friction between the hill and the sled is 0.212 and the coefficient of static friction between the hill and the sled is 0.317. How hard does the father need to pull the sled to keep it moving at a constant velocity? The child holds on tightly and so doesn't fall off the sled. The hill makes an angle of 15.5° with the horizontal.



$Note\ that: \theta = 15.5° \qquad m = (22 + 3.2) = 25.2kg \qquad g = 9.8m/s^2$

$First, find\ the\ components\ of\ the\ \vec{F}_g\ vector: \vec{F}_{g,x} = F_g \sin\theta = mg\sin\theta \quad \& \quad \vec{F}_{g,y} = F_g\cos\theta = mg\cos\theta$

$In\ the\ y\text{-}direction, there\ is\ no\ motion, as\ the\ sled\ does\ not\ lift\ off\ the\ snow. Thus: \vec{F}_{net,y} = 0$

$\vec{F}_N = \vec{F}_{g,y} = mg\cos\theta$

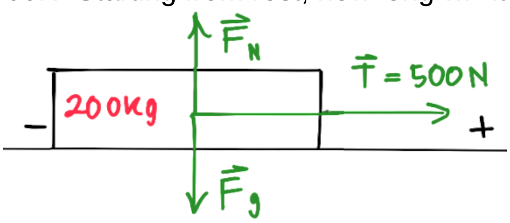
$In\ the\ x\text{-}direction, there\ is\ no\ acceleration, as\ the\ sled\ is\ moving\ at\ constant\ velocity.$

$Hence\ according\ to\ Newton's first\ law\ of\ motion: \vec{F}_{net,x} = 0$

$\vec{F}_{pull} = \vec{f}_k + \vec{F}_{g,x} \quad \rightarrow \quad \vec{F}_{pull} = \mu_k \vec{F}_N + mg\sin\theta = 0.212(mg\cos\theta) + mg\sin\theta$

$= mg(0.212\cos\theta + \sin\theta) = (25.2 \times 9.8)(0.212\cos 15.5° + \sin 15.5°) = 116.45N$

EXAMPLE 4: A horizontal cable pulls a 200kg cart along a horizontal track. The tension in the cable is 500N. Starting from rest, how long will it take for the cart to read a speed of 8m/s?



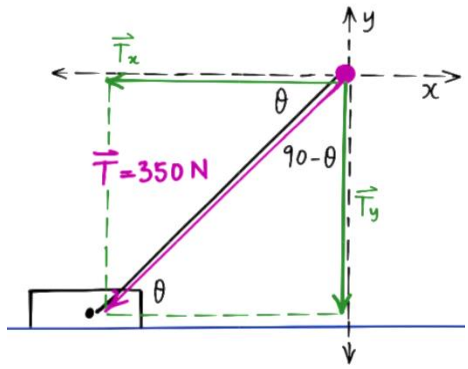
$F_{net} = T = 500N$

$a = \dfrac{F_{net}}{m} = \dfrac{500N}{200kg} = 2.5m/s^2$

$v_f = v_i + at \rightarrow 8 = 0 + 2(2.5)t \rightarrow t = \dfrac{8}{2.5} = 3.2s$

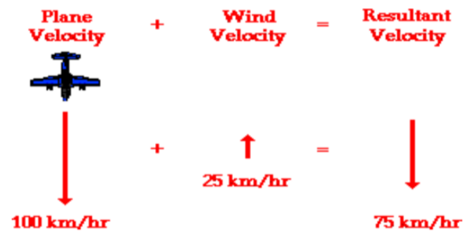
EXAMPLE 5: Helen is parasailing. She sits in a seat harness which is attached by a tow rope to a speedboat. The rope makes an angle of 51° with the horizontal and has a tension of 350N. Determine the horizontal and vertical components of the tension force.

$T_x = 350\cos\theta = 350\cos 15°$
$T_y = 350\cos(90-\theta) = 350\sin\theta = 350\sin 15°$

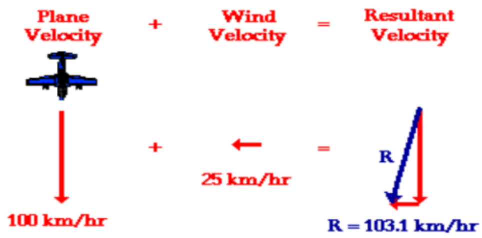


RESULTANT VELOCITY

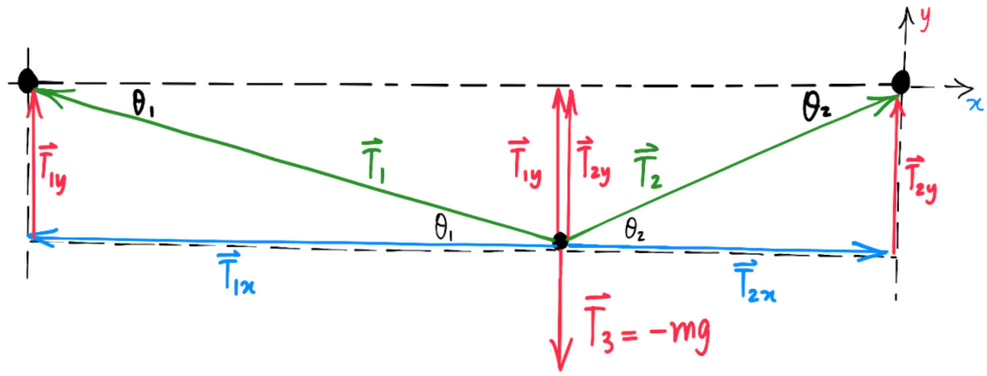
A plane travelling with a velocity of 100km/hr south meets a ***HEADWIND*** with a velocity of 25km/hr north. The resultant velocity would be 75km/hr relative to an observer on the ground.



A plane travelling with a velocity of 100km/hr south meets a ***SIDEWIND*** with a velocity of 25km/hr east. The resultant velocity would be the vector sum of the two individual velocities.



EXAMPLE 6: A rope hangs between 2 poles. A 90N boy hangs from it, so that one side of the rope makes an angle of 5° to the horizontal, and the other side makes an angle of 10° to the horizontal. Work out the tension in each side of the rope.



$Given\ \theta_1 = 5°\ \&\ \theta_2 = 10°$
$To\ calculate\ components:$
$T_{1x} = T_1\cos\theta_1 \quad \& \quad T_{1y} = T_1\sin\theta_1 \qquad\qquad T_{2x} = T_2\cos\theta_2 \quad \& \quad T_{2y} = T_2\sin\theta_2$

$In\ the\ y\text{-}direction, given\ that\ there\ is\ no\ motion:$
$T_{1y} + T_{2y} - T_3 = T_{1y} + T_{2y} - mg = 0\ \ Thus \rightarrow\ T_1\sin\theta_1 + T_2\sin\theta_2 - 90 = 0 \rightarrow T_1\sin 5° + T_2\sin 10° = 90\ [1]$

$In\ the\ x\text{-}direction, given\ that\ there\ is\ no\ motion:$
$T_{2x} - T_{1x} = 0 \rightarrow T_2\cos\theta_2 - T_1\cos\theta_1 = 0 \rightarrow T_2\cos 10° - T_1\cos 5° = 0\ [2]$
$Simultaneously\ solve\ [1]\ \&\ [2]\ to\ find\ T_1\ \&\ T_2$