**Six Step Hypothesis Testing**

1. Set up null and alternate hypotheses
   a. One tail
   b. Two tail
2. Determine test statistic and sampling distribution
3. Specify significance level
4. Define decision rule
5. Compute value of test statistic and corresponding p-value
6. Make a decision and conclude

**Hypothesis Testing for normal distribution**

Null Hypothesis

When X is normally distributed, SK = 0 and K = 3

$H_0$ : X is normally distributed

Reject $H_0$ if $\widehat{SK}$ is significantly different from zero or/and $\widehat{K}$ significantly different from 3

*Skewness*

$$SK = \frac{E(X-\mu)^3}{\sigma^3}$$

$$\widehat{SK} = \frac{1}{n}\sum_{i=1}^{n}(\frac{X_i - \bar{X}}{s})^2$$

*Kurtosis*

$$K = \frac{E(X-\mu)^4}{\sigma^4}$$

$$\widehat{K} = \frac{1}{n}\sum_{i=1}^{n}(\frac{X_i - \hat{X}}{s})^4$$

*Jarque-Bera statistic*

$$JB = \frac{n}{6}(\widehat{SK}^2 + \frac{(\widehat{K}-3)^2}{4})$$

When $H_0$ is true, $JB \sim \chi^2_{(2,\alpha)}$, at specified significance level, reject $H_0$ and conclude X is not normally distributed if JB ≥ $\chi^2_{(2,\alpha)}$.

**Tests for comparing two populations**

*Random and independent sample*

*Known variances*

Test Statistic (Z-Stat)

$$Z = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_1^2}{n_2}}} \sim N(0,1)$$

*Variances unknown and unequal*

Test Statistic (t-stat)

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \sim t_{(n_1+n_2-2)}$$

*Variances unknown and equal*

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{s_p^2(\frac{1}{n_1} + \frac{1}{n_2})}} \sim t_{(n_1+n_2-2)}$$

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

**Testing equality of two population variances using independent normally distributed samples**

Test statistic

$$\frac{(n_1 - 1)s_1^2}{\sigma_1^2} \sim \chi^2_{(n_1-1)}$$

$$F = \frac{\chi^2_{(n_1-1)}/(n_1 - 1)}{\chi^2_{(n_2-1)}/(n_2 - 1)} = \frac{\frac{(n_1 - 1)s_1^2}{\sigma_1^2}/(n_1 - 1)}{\frac{(n_2 - 1)s_2^2}{\sigma_2^2}/(n_2 - 1)} = \frac{s_1^2/\sigma_1^2}{s_2^2/\sigma_2^2} \sim F_{(n_1-1,n_2-1)}$$

When $H_0: \sigma_1^2 = \sigma_2^2$ is true, $F = \frac{s_1^2}{s_2^2} \sim F_{(\alpha,n_1-1,n_2-1)}$

Decision rule

Reject H$_0$ if $F > F_{(\alpha,n_1-1,n_2-1)}$ or if $F < F_{(1-\alpha,n_1-1,n_2-1)}$

$$F_{(1-\alpha,v_1,v_2)} = \frac{1}{F_{(\alpha,v_1,v_2)}}$$