FNCE30001 – Investments (Equity)

Lecture 1- Risk Aversion and returns

Risk, uncertainty, return

Investment: a trade-off between present and future benefits. Must compensate investor for: (1) the time the funds are committed, (2) the expected rate of inflation and (3) the uncertainty of the future payments.

Holding Period Return:

appreciation ↘ yield ↙

$$\textbf{\textit{Gross Return}} \qquad R_{t+1} = \frac{P_{t+1} + C_{t+1}}{P_t} = \frac{P_{t+1}}{P_t} + \frac{C_{t+1}}{P_t}$$

P_t = buying price. P_{t+1} = selling price. C_{t+1} = dividend/payment received (coupon, rent, interest).

***Note:** this is never negative, 0 means all wealth is lost, 1 means no change in wealth, >1 means wealth is increased.

$$\textbf{\textit{Net Return}} \qquad r_{t+1} = R_{t+1} - 1$$

***Note:** can be negative. -1 means all wealth is lost, 0 means no change in wealth, >0 means wealth is increased.

E.g.

Gross return = (104 + 1)/100 = 1.05. Net return = (104 + 1)/100 – 1 =0.05.

$$\textbf{\textit{Simple Return}} \qquad r_{t+1} = \frac{P_{t+1}}{P_t} - 1$$

***Note:** i.e. simple return = gross or net return

$\textbf{\textit{Log Return}} \qquad \overline{r}_{t+1} = \ln(1 + r_{t+1}) = \ln(R_{t+1})$

***Note:** when r_{t+1} is close to 0, then the equation comes closer to ln(1) which = 0.

Multiple period returns:

Simple interest: $W_2 = W_0(1 + r_1 + r_2)$. E.g. 100(1+0.05+0.1) = 115

***Note:** W_2 means wealth at end of period 2, W_0 means wealth at end period 0.

Compound interest: $W_2 = W_0(1 + r_1)(1 + r_2)$. E.g. 100(1+0.05)(1+0.1) = 115.50

Notation: $R_{t,t+N} = \frac{P_{t+N}}{P_t}$, $r_{t,t+N} = R_{t+N} - 1$, $\overline{r}_{t,t+N} = \ln\left(\frac{P_{t+N}}{P_t}\right) = \sum_{i=1}^{N} \overline{r}_i$

Real and nominal returns:

Use real numbers with real returns and nominal numbers with nominal returns (DON'T MIX UP!)

$$r_{t+1}^{real} = r_{t+1}^{nominal} - i_{(t+1)}$$

E.g. 1 year ago, deposited $1000 in bank 10% return. If i = 0.06, then real return = 10% – 6% = 4%

The higher the expected inflation, the higher the required return on a risky asset.

Comparing returns across periods:

Use the EAR (effective annual rate) $= \left(1 + r_{t,t+n}\right)^{\frac{1}{n}} - 1.$ ***Note:** n is the period the investment is held for (in years), $r_{t,t+n}$ = net return.

Sometimes we want to know how well our investment performed on 'average'…so we use:

Arithmetic mean return: this is not an appropriate method for calculating an average because they are not independent. i.e. if you lose 100% of your capital in one year, you don't have any hope of making a return on it during the next year so they require a geometric average to represent their mean.

$$\mu_a = \frac{1}{n}\sum_{i=1}^{n} r_{t+i} = \frac{1}{n}(r_{t+1} + r_{t+2} + \ldots + r_{t+n-1} + r_{t+n})$$

Geometric mean return: the annual rate that will compound to the observed terminal value of a portfolio.

$$\mu_g = \left(\prod_{i=1}^{n}(1 + r_{t+i})\right)^{\frac{1}{n}} - 1 = \Big(\underbrace{(1 + r_{t+1}) \times \ldots \times (1 + r_{t+n-1})(1 + r_{t+n})}_{R_{t,\,t+n} = P_{t+n}/P_t}\Big)^{\frac{1}{n}} - 1$$

E.g. $P_2 = P_0(1+0.05) = P_0(1+\mu_g)^2 \therefore (1+\mu_g)^2 = P_2/P_0 = R_{t,\,t+n} \therefore \mu_g = (R_{t,\,t+n})^{1/2} - 1$

Expected return:

$$E(r) = \sum_{s=1}^{n}(Probability\ of\ Scenario) \times (Possible\ Return) = \sum_{s=1}^{n} p_s \times r_s$$

BUT: because we can't observe the population we must rely on a sample of returns.

Sample mean: $E(r) = \frac{1}{n}\sum_{i=1}^{N} r_i = \bar{\mu}$ This provides an estimate of the E(r).

So… $E(r) = \sum_{s=1}^{n} p_s \times r_s = \frac{1}{n}\sum_{i=1}^{N} r_i = \bar{\mu}$

Variance:

$$Var(r) = \sum_{s=1}^{n} p_s \times (r_s - E(r))^2 = E[(r_s - E(r))^2]$$

***Note:** the standard deviation $Stdev(r) = \sqrt{Var(r)}$ measures how well the return of an asset compensates the investor for the risk taken.

Sample variance:

Sample variance is biased downward because we have taken deviations from the sample average $\bar{\mu}$ instead of the unknown, true expected value E(r), and so have introduced a bit of estimation error. We can eliminate the bias by multiplying the sample variance by $\frac{n}{n-1}$.

$$\therefore V(r) = \frac{n}{n-1} \times \frac{1}{n}\sum_{i=1}^{N}(r_i - \bar{\mu})^2 = \frac{1}{n-1}\sum_{i=1}^{N}(r_i - \bar{\mu})^2$$

Sharpe ratio:

$$S = \frac{E(r) - r_f}{\sigma_r}$$

The ratio measures the excess return (or risk premium) per unit of deviation in an investment asset.

***Note:** we want the outcome which maximises the Sharpe ratio (steepest gradient on line).

Defining return

- r1 is the return from t = 0 to t = 1, given by:
  - $r_1 = \frac{P1+D1-P0}{p0} = \frac{P1+D1}{P0} - 1$
- The return is the ex-dividend price (P1) plus any dividend (D) received at t = 1 , in excess of the price paid (P0), as a fraction of the price paid.
- When the time frame is clear, we may omit the subscript, so return is just r.
- A return may be:
  - The unknown future return $\tilde{r}$:
  - The return that's "expected": $E(\tilde{r})$
  - The realised return: r
- The tilde ($\tilde{}$) indicates a random variable; i.e one that has a probability distribution.
- Returns are uncertain because there are many things we don't know
- We usually start by tying investments to the economy:
  - What can happen to the economy in the future (each outcome or "state" s)?
  - What return will Qantas get in each outcome $\tilde{r}(s)$
  - How likely is that outcome p(s)?
- **Outcomes must be mutually exclusive and exhaustive.**
- **Probabilities must sum to 1**

Expected return

- $E(\tilde{r}) = \sum_s p(s)\tilde{r}(s)$

Standard deviation of returns

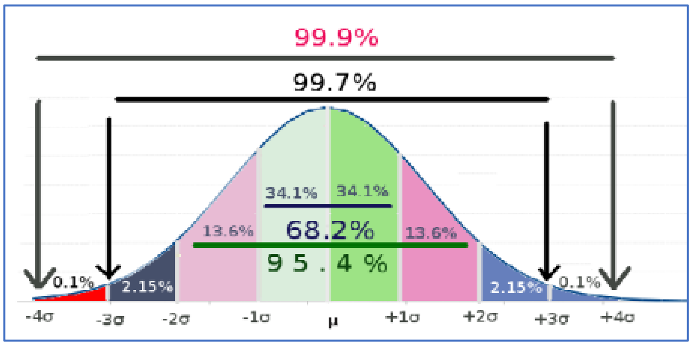
- The standard deviation of the return distribution is a measure of the uncertainty of returns.
- Also called the "volatility" of returns.
- The variance (SD^2) of returns is:
  - $\sigma^2(\tilde{r}) = \sum_s p(s)[\tilde{r}(s) - E(\tilde{r})]^2 = E\left[\left(\tilde{r} - E(\tilde{r})\right)^2\right]$
- Therefore, SD:
  - $\sigma(\tilde{r}) = \sqrt{\sum_s p(s)[\tilde{r}(s) - E(\tilde{r})]^2}$

Modelling Returns

- What distribution might make a good model for $\tilde{r}$
- The Normal, with mean μ and volatility σ has these benefits:

- **Tractable:**
  - Symmetric
  - Only two parameters needed, so σ is an appropriate measure of risk.
- **Stable:**
  - If asset returns are normally distributed then portfolio returns will also be normally distributed.
- **Empirically, a reasonable first approximation**

- Computing confidence intervals and significance tests is easy:



- Often, it is useful to scale returns so they match the standard normal, with mean 0 and standard deviation 1: that is, N(0,1)
- Suppose $\tilde{r}_t \sim N(\mu, \sigma)\ Then$:
  - We can transform it into a standard normal random variable $\tilde{Z}$
  - $\tilde{Z} = \frac{\tilde{r}_t - \mu}{\sigma}$
- We can then compute probabilities using (eg) a standard normal table

Suppose you want $\Pr(\tilde{r}_t \leq 0)$.

$$\text{Then } \Pr(\tilde{r}_t \leq 0) = \Pr\left(\frac{\tilde{r}_t - \mu}{\sigma} \leq \frac{0 - \mu}{\sigma}\right)$$

$$= \Pr\left(\tilde{z}_t \leq \frac{0 - \mu}{\sigma}\right)$$

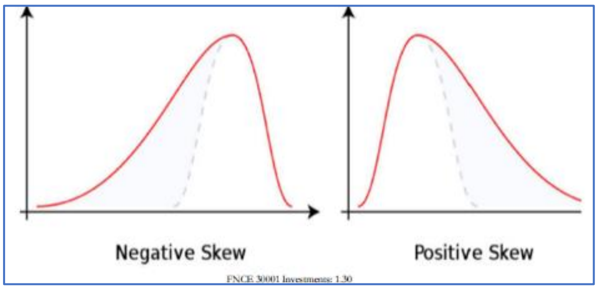
where $\frac{0 - \mu}{\sigma}$ is the "critical value".

- Returns however, are not in fact normally distributed, especially over short time periods like a day or a week.
- A t-distribution with 5 df tends to be the closest approximation

Skewness

- The normal distribution is symmetric. (equal to 0)
  - But many other distributions are not.
  - They may be “skewed”.

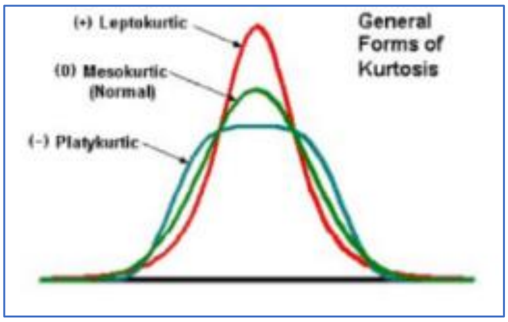
- Skewness is defined as:
    - $Skew = \frac{E(\tilde{r}-\mu)^3}{\sigma^3}$



- If <u>returns are negatively skewed</u> (ie to the left) then
    - downside risk is underestimated by the standard deviation.
- If <u>returns are positively skewed</u> (ie to the right) then
    - downside risk is overestimated by the standard deviation.

Kurtosis

- Kurtosis indicates how "fat" the tails are.
- Excess kurtosis is defined as:
    - $Kurtosis = \frac{E(\tilde{r}-\mu)^4}{\sigma^4} - 3$
- Note: the kurtosis of the normal distribution is 3.
- Unfortunately, some people use the term "kurtosis" when they really mean "excess kurtosis".



- <u>"Platykurtic"</u>
    - Negative excess kurtosis
    - Tails have less data (ie are "thinner") than the normal.
- <u>"Leptokurtic"</u>
    - Positive excess kurtosis
    - Tails have more data (ie are "fatter") than the normal.

Conclusions

1. Since the 1950s, average skewness has been tiny
    - And has been unstable from decade to decade.

2. But average excess kurtosis has been positive (about 0.4).
   - And reasonably stable from decade to decade
   - ie there have been "fat tails".

Portfolio Theory

- "Portfolio": Any collection of investments (assets).
- "Modern Portfolio Theory" was invented by Harry Markowitz (born 1927) in the 1950s.
  - It won him the Nobel Prize in Economics in 1990.
- Before that time, most investors intuitively understood its message.
  - But Markowitz gave it a rigorous foundation.
  - And made its implementation more efficient.
- Portfolio theory is a foundation for both the theory and practice of Finance.
- While there have been numerous refinements to portfolio theory in the past 60 years, the basics are still very like those laid down by Markowitz himself.

2 ways to control the risk of a portfolio

- Shift funds between risky assets and the risk-free asset:
  - **capital allocation**
- Shift funds between risky assets:
  - **efficient diversification**
- Both problems have two elements:
  - The risk-return combinations available:
    - **"objective" element**
  - The risk-return preferences of the investor:
    - **"subjective" element**

Capital Allocation

- about splitting funds between safe and risky assets
  - Safe: eg a Treasury note or a government guaranteed bank deposit
  - Risky: eg a share or a portfolio of shares
- "Investment opportunity set":
  - The risk-return combinations available to the investor
- Investor puts proportion y of his/her wealth in the risky portfolio and the remaining 1 – y in the risk-free asset.
- This creates the complete (or, combined) portfolio C.
- Note:
  - $proportion\ y = \frac{\sigma_C}{\sigma_p} = \frac{risk\ of\ combined\ portfolio}{risk\ of\ risky\ asset\ portfolio}$
- The capital allocation line (CAL) is the graph of the investment opportunity set.
  - ie the set of feasible pairs of expected return and standard deviation
- Its equation is:
  - $E(\tilde{r}_c) = r_f + \frac{E(\tilde{r}_p) - r_f}{\sigma_p}\sigma_c$
  - Intercept: r_f