FNCE30001 Course Summary Notes

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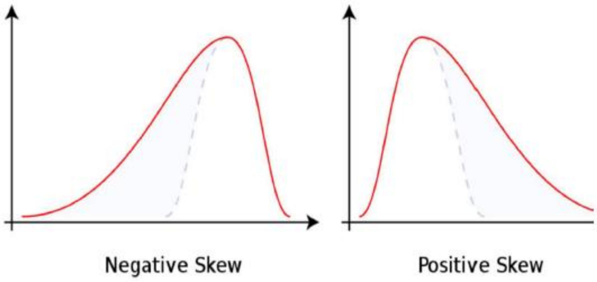
Returns

- r_1 is the return from t=0 to t=1, which is: $r_1 = \frac{P_1 + D_1 - P_0}{P_0}$
  - $\tilde{r}$ – unknown future return (tilde indicated variable with probability distribution)
    - $\tilde{r}$ (s) – return in state 's', which has probability of occurring: p(s)
  - E($\tilde{r}$) – expected future return = $\sum p(s)\, \tilde{r}$ (s)
  - r – realised (actual) return
- Variance (σ^2) = $\sum p(s)\, [\tilde{r}(s) - E(\tilde{r}(s))]^2$ = $\sum E[\tilde{r} - E(\tilde{r})]^2$

Distribution of returns

- Returns can be modelled to the **'normal distribution'**
  - $\tilde{z} = \frac{\tilde{r}_t - \mu}{\sigma}$, where $\tilde{r}_t \sim N(\mu, \sigma)$
    - Can then calculate probabilities using a standard normal table or Excel
    - Can approximate returns in a normal distribution by the '68.2, 95.4, 99.7 rule'
  - NB: natural logarithm returns are closer to being accurate, but have too "fat tails"
- Skewness – *'which way it leans'*
  - Skew $= \frac{E(\tilde{r} - \mu)^3}{\sigma^3}$
  - Positive skew – less negative tail
    - Stock market is this (as cannot go below $0 value), meaning a normal distribution **over-estimates** downside risk
  - Negative skew – less positive tail
  - NB: skewness decreases when considering data over time – has been tiny since 1950's



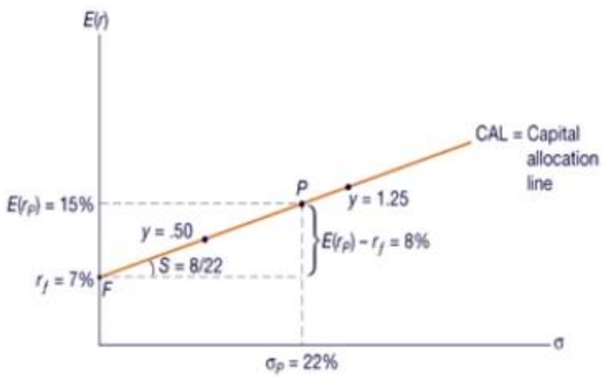
- Kurtosis (K) – *'how fat the tails are'*
  - Normal distribution = 3
  - Platykurtic: K<3 (fatter in middle, thinner tails)
  - Leptokurtic: K>3 (thinner in middle, fatter tails)
  - Excess kurtosis (any kurtosis over 3) $= \frac{E[(\tilde{r} - \mu)^4]}{\sigma^4} - 3$
    - Average excess kurtosis approx. 0.4 since 1950's

Portfolio Theory

<u>Capital Allocation Line (CAL)</u>

The balance between risky ('y' proportion) and risk-free ('1 – y' proportion) investments

- CAL considers all the available risk-return combinations
- $E(\tilde{r}) = r_f(1 - y) + yE(\tilde{r}_p)$
  - 'y' can be >1 (i.e. borrowing at risk-free rate and investing in risky portfolio)
- $\sigma_{combined\ portfolio} = y\sigma_p$ (only includes risky, as risk-free has σ=0)
- CAL plots $E(\tilde{r})$ against σ, for given values of 'y'
  - $E(\tilde{r}) = r_f + \frac{E(\tilde{r}_p) - r_f}{\sigma_p}\sigma_{combined}$
- Capital Market Line (CML)
  - A special case of the CAL, when P=M (the "market portfolio")



Sharpe ratio (a.k.a. 'reward-to-volatility' ratio)

- The rate at which return increases when risk (σ) increases – the **gradient of the CML**

- So **all the points on the CAL have the same Sharpe ratio**
- $S_p = \frac{E(\tilde{r}_p) - r_f}{\sigma_p}$
  - $E(\tilde{r}_p)$ = return on **risky assets only**
- Higher Sharpe ratio indicates better performance

Utility function

- Represents an investor's attitude to risk
  - Determine where along the CAL the investor will fall (a retrospective measurement, not practically useful)
  - Aim to **maximise** utility (i.e. make U as big as possible)
- $U_i = E(\tilde{r}_i) - \frac{1}{2} A \sigma_i^2$
  - $E(\tilde{r}_i)$ = expected return of **combined** (risky and risk-free) **portfolio**; σ_i^2 = variance on combined portfolio
  - Risk averse: A > 0
  - Risk neutral: A = 0 (irrelevant)
  - Risk seeking: A < 0 (irrelevant)

Qn1. Given the following portfolio options, with a risk-free rate of 4%, which would an investor with A = 2 choose?

Portfolio	Risk Premium (%)	Expected return (%)	Risk (SD) (%)
Low risk	4	8	6
Medium risk	6	10	12
High risk	10	14	19

Ans: $U_i = E(\tilde{r}) - \frac{1}{2} 2 \sigma_i^2 = E(\tilde{r}) - \sigma_i^2$
Low risk: $U_i = 0.04 + 0.04 - 0.06^2 = 0.0764$
Medium risk: $U_i = 0.06 + 0.04 - 0.12^2 = 0.0856$
High risk: $U_i = 0.10 + 0.04 - 0.19^2 = 0.1039$
Utility is maximised with the high risk portfolio, therefore the investor would invest in that one

- Selecting 'y' so as to maximise utility
  - Substituting in $E(\tilde{r}) = r_f + y[E(\tilde{r}_p) - r_f]$ to $U_i = E(\tilde{r}) - \frac{1}{2} A \sigma_i^2$ gives:
    - $U_i = r_f + y[E(\tilde{r}_p) - r_f] - \frac{1}{2} A \sigma_i^2$
    - To get the maximum utility, we differentiate and solve = 0, which gives:
      - $y^* = \frac{E(\tilde{r}_p) - r_f}{A \sigma_p^2} = \frac{S_p}{A \sigma_p}$

Qn2. Given $r_f = 4\%$, $E(\tilde{r}_p) = 18\%$ and $\sigma_p = 25\%$, what proportion would an investor with A = 5 put into the risky portfolio?

Ans: $y^* = \frac{E(\tilde{r}_p) - r_f}{A \sigma_p^2} = \frac{0.18 - 0.04}{5 \times 0.25^2} = 0.384$
So they would put 38.4% of their funds in the risky portfolio, and the remaining 61.2% in the risk-free asset
$E(\tilde{r}_{combined}) = r_f + y[E(\tilde{r}_p) - r_f] = 0.04 + 0.384[0.18 - 0.04] = 10.61\%$
$\sigma_{combined} = y \sigma_p = 0.384 \times 0.25 = 9.6\%$
$S_{combined} = \frac{0.1061 - 0.06}{0.096} = 0.48$ (which = S_P, proving **all the points on the CAL have the same Sharpe ratio**)

<u>Optimal portfolio choice</u>

Diversification (which risky asset, and how much in each)

- Sources of risk
  - Market (**systematic**) risk