#1 - DS – FUNDAMENTALS (Lecture 1)

Derivative security(contract)
Financial security whose value is determined by the value of something else called the underlying variable/asset
4 main types: options, forwards & futures, swaps, credit derivatives
Price of a traded asset: stock, stock index, bond/interest rate, currency, commodity, another derivative.

Market, exchange over the counter, standardised (amt, maturity), OTC – private, specifically tailored)

Derivatives - HUGE (hundred of trillion $, gross market value - $20 trillion)

- Underlying asset x famous, over complicated
- Interest rate (75%), currencies (15%)

Hedger – has an exposure to price of underlying ↓ , cr8 D for derivatives.
Price fall, buy put or sell forward

Speculator – take risk for profit.
Stock rise – buy call/buy forward

Attractive – leverage an investment without having to borrow funds.

Arbitrageur – making a profit without risk.
Derivatives – increase/decrease risk – dangerous if misused

Option – choice & flexibility to do something, flexibility has value – make decision later rather than now

Financial Option – agreement one party the right but not obligation to ex8 specific asset with the other party @ specific price (strike, exercise) @ a later date.

Buy – long // Sell – short (writing option)

Long decides – short comply
Option valuable so long as pays premium to short
Long (premium -> short // <- option

Last date – maturity/ expiry (T)

European @ Maturity
American any time (highest)
Bermudan – several times.

Option – call long right but not obligation to buy stock from short at time of exercise

Put – long right but not obligation to sell stock at time of exercised.
Counterparty risk – one party defaults
Derivatives market – minimise counterparty risk (margin system, collateral requirements & clearing houses, important during times of crisis.

Not exercised @ Maturity – all contractual rights & obligations cease.

European call option, Strike = $7, T =1, So = $8 , price of call is $1.5 buy one call

ST(receive)	Pay (X)	Value of ex8 (ST-X)	Value of option to exchange (CT) – avoid loss	Return on option (CT-1.5/1.5)	Return on stock (ST-8/8)
5	7	-2	0	-100	-37.5%
7	7	0	0	-100	-12.5%
8	7	1	1	-33.5%	0%
11	7	4	4	+166.66%	37.5%

Put, strike = $20, T=1 , So = $25, Price of put $2

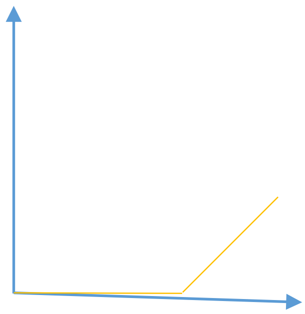
ST(receive)	Pay (X)	Value of ex8 (X-ST)	Value of option to exchange (CT) – avoid loss	Return on option (CT-2/2)	Return on stock (ST-25/25)
17	20	3	3	50%	-32%
19	20	1	1	-50%	-24%

20	20	0	0	-100%	0%
30	20	-10	0	-100%	20%

Payoff of option @ maturity – whether exercise depend on value @ maturity.
Value of option & decision to exercise determined simultaneously, X & St
Long has choice, exercise positive value, zero sum gain

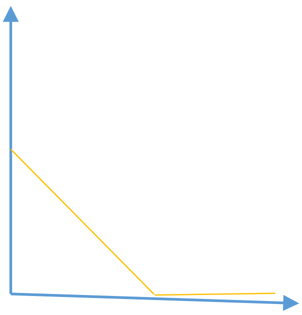
Position	Value of option at maturity if ST<X	ST>X	Exercise if option	Value @ maturity
Long call	0	ST-X	ST>X	Ct = max (0, St-x)
Short call	0	-(ST-X)		-CT
Long put	X-St	0	ST<X	Pt = max (0, X-ST)
Short put	-(X-St)	0		-PT

Plot – payoff diagram.



X St

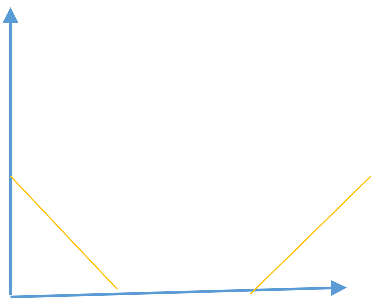
Long Call



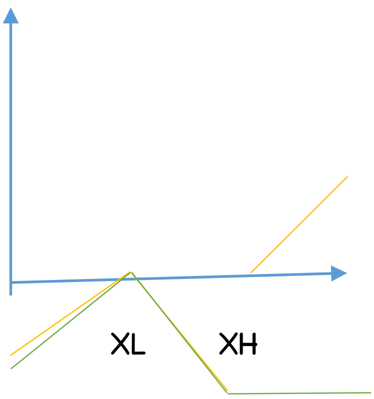
X St

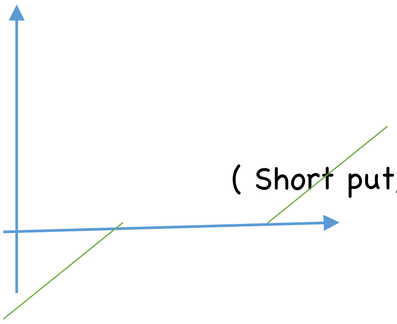
Long Put

Long [call + put] = X care what happens to underlying asset
Short[call + put] = in middle



XL XH





XL XH

(Short put, short call, long two call)

Profit – payoff paid at time contract entered into
Profit long = payoff-premium
Profit short = payoff + premium

(ignore time value of money) - plot of profit, profit diagram
Exercise depend payoff not profit

Cash inflow – positive ☺
Cash outflow – negative ☹

1. Compounded once per annum, future value FV = a $(1+r)^T$
2. Compounded m per annum, future value of investment at end of T years : FV =$(1 +R/M)^{mT}$
3. m- infinity, r continuously compounded FV = Ae^{rt}
4. Continuous compounding – XE^{rt} future value of X, Xe^{-rt} present value

Short sell – buy asset (long) before sell , can sell before buy
Short – borrow asset from someone, buy asset later to repay borrow , pay lender fee for borrow asset
-Income is paid on asset during period of short sell, short seller pay = amount, income lender entitled have received by party whom asset sold

Cash flow from short selling opposite from buying asset

	T=0	Div date	T
Long stock	-So	+dt	+St
Short stock	+So	-dt	-St

Long stock – gain price rise / lose price fall
Short stock –gain price fall/ lose price rise
Short selling – speculative, hedging/ arbitrage purposes

Short selling X dollars of riskless, T – year 0 CB is equivalent to borrowing X dollars for T-years at rf rate

	T=0	T

Short bond	+x	$-xe^{rt}$
Borrowing	+x	$-xe^{rt}$

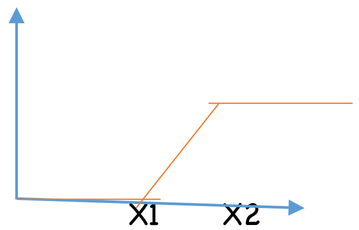
Combined – wide range of payoff & profit to suit – hedging & speculative
Protective put – long put long stock
Covered call- short call and long stock
Straddle – long call long put on same stock same x,T – 1^{st} picture (trade volatility – big/small news)
Spread – two+ calls on same stock diff X same T (move strike prices)
Payoff- building block, determine payoff, pay off on combination – sum of payoff

European callx & T, sell one European call strike x and t for price c2 when x1<x2

	ST<X	X<ST<X	ST>X
Buy 1 call strike x1	0	St-x	St-x
Sell 1 call strike	0	0	=(st-x)
Payoff	0	St-x	X2-X1

Premium c2-c1

Bull spread using calls



#2 – NO-ARBITRAGE (Lecture 2)

Arbitrage – trading strategy – cost nothing but e(x) cash inflow later with no chance cash outflow later. – free lunch

Trading strategy	Cash Now	Cash Later
Buy shares now & sell them later(xA)	-	+
Short sell now and buy them back later(xA)	+	-
Arbitrage strategy I	0	+
Arbitrage strategy II	0	+ (only some state of world & 0 in all other states)
Arbitrage strategy III	+	0

Arbitrage strategy IV	+	+

No negative cash flow & some chance of positive cash flow.

	T=0	T=1
ANZ	100	-103
CAN	-100	104
	0	1

	T=0	T=1
ANZ	100	-103
CAN	-103/1.04 =99.04	103
	.96	0

Trading – sell (high price), buy (low price)

Law of One Price (LOOP)

- X is Xo & Y is Yo
- CF on X is X1 & Y – Y1

If XI=Y1 , X0 = Y0 , arbitrage, identical cash flow in future = same price today

X0>YO, arbitrage – high sell X, low buy Y

Action	Cash now t=o	Cash later, T=1
Buy 1 Y	$-Y0$	+Y1
Sell 1 X	+Xo	-X1
	X0-YO > 0	Y1-X1 =0

Loop – relative price 2+ assets, arbitrageurs selling pressure x (down price, buying pressure Y(up price) – equate. LOOP – multiple cash flows.

Arbitrage- fundamental financial market , rare – well functioning financial market, arbitrage- disappear quick readjustment.

Assumption of no-arbitrage;

Valuation by replication

LOOP – copy fcf on one asset use fcf on some other asset – current value first = second , if not arbitrage

Bond a – 1000 in t=1 ; p = 990

Bond b – 1000 in t = 2 ; p = 960.79

Bond c – 100 in t=1, 1000 in t=2

- Buy 1 bond a, 10 bond b , 10 bond c

	T=0	T=1	T=2
Buy 1 Bond A	-990	1000	0
Buy 10 Bond B	-9607.90	0	10000

Net cash flow	-10597.90	1000	10000
Buy 10 Bond C	-10P	1000	10000
Net cash flow	-10P	1000	10000

Loop – prices same if not arbitrage
10P = 10597.90 ; P = 1059.79 P = 1080 , sell bond c , c = 1040 , too low (buy c – sell a & b – 1060, profit 10 .

Arbitrage – expect a gain(chance of +ve CF) with no risk (-ve cf) =price 2 asset – risk

Put call parity – price of European call & price of European put on same stock, strike, maturity no div

	T=0	T St<x	T=2 St>x
Long call	-Ce	0	St-x
Short Put	+P	-(X-ST)	0
Net	P-c	St-x	St-x
Long stock	-So	St	St
Short bond	XE^{-RT}	-x	-x
Net	XE^{-RT}-So	St-x	St-x

Loop = P-C = XE^{-RT}-So

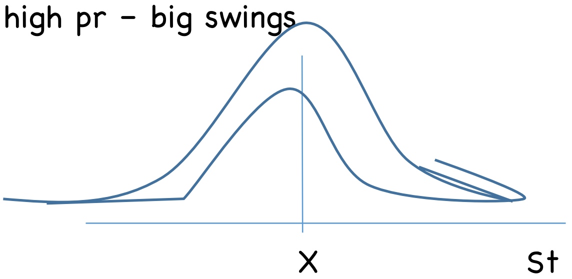
Call = put + stock – bond, pcp no hold – opportunity for arbitrage

c> p + s - XE^{-RT}

Too high, sell call , rhs low so buy
Buy put , buy stock buy – bond (-ve amount – sell bond)
Trading to capture arbitrage profit – sell call, buy put , buy stock, sell bond

Value of option prior to maturity

Value of option at maturity – ST & X
Prior to maturity- S0, x, time, risk free interest rate, div, volatility (dispersion)
high pr – big swings



Keep RHS, dispersion – options accept gains

St – up (call), down (put) ; BAD NEWS, DIVIDEND

Volatility over remaining life – estimate

Increase one variable others constant

Variable	European Call	European Put	Intuition
Stock price	+	-	Higher stock price 18
Dividends	-	+	Stock price fall, div paid, lower stock price
Strike	-	+	Pay more exercise of call receive more exercise of put
Risk free	+	-	Pay less in terms on exercise call – receive less put in PV exercise of put
Volatility	+	+	Higher pr – higher payoff
Time	?	?	

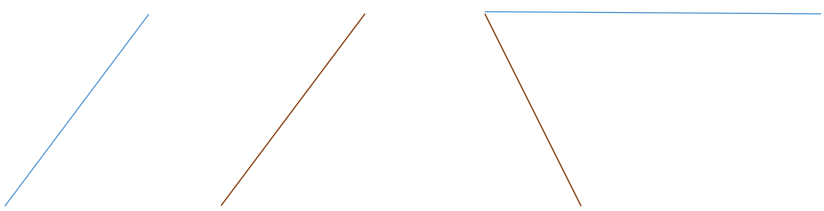
Time – good / pv.
Bull spread using call
American calls and puts time + (more time whether to exercise)

Moneyness (long)

1. In the money – gain exercised @ that time
2. At the money – indifferent exercised @ that time
3. Out of the money-lose exercise @ that time.

Intrinsic value/Time value
Intrinsic – option = maximum of zero & value have exercised @ that time.
Time value – option = option differ from intrinsic value

Arbitrage bounds on option prices – sensible price range for call & put
Market price of option fall outside range- arbitrage available.
European call with no div- remaining life of option
S>c> max (0, $S0-XE^{-RT}$)



x

$XE^{-RT} > P > (\max 0, xe^{-rt} - S0)$

Option price fall outside bonds
17 – European call, stock option – 2.8 – underlying stock 20, rf =2

$20 = So > Ce > \max (0, S- XE^{-RT}) = 3.34$
Fall below lower bound, arbitrage

	Cash flow now t=0	Cash flow at maturity S<17	S>17
Buy call	-2.8	0	0
Sell stock	+20	-st	-st
Buy bond	-16.66	17	17
Net cash flow	0.54	17-st	0

Stock – dividend, ex-div remaining,
$PV(d) = de\text{-}^{rt}$

European call and put

$So - PV(D) > C > \max (0, S\text{-}PV(D)\text{-}XE^{-RT}$
$XE^{-RT} > P > \max (o, pv (d) + XE^{-RT} - S$

American – any time , exercised early –
Depends : option call/put, dividend over life.

Without div- wait to pay money rather than pay money today.
Put- paid money now than later
Giving up dividend, give up flexibility exercise later & time value of money

American – cannot be worth less/ equivalent
Diff in value – early exercise premium
Optimal early exercise change arbitrage

Low price share – pay fixed price , how big div
Put – wont exercise till after dividend drop

Div & early exercise pcp
Arbitrage, alternative, inequality for American
American Option

Call	X Div	Never exercise early	CA=CE	Lower bound: Ca> max (0, s-xe^{-rt}
Call	Yes div	Prior to ex div date	Ca>CE	CA> max (0, S-PV(D) - XE^{-RT}, S-X)
Put	X div	Maybe	Pa>PE	PA> MAX (0,X-S)
Put	Yes	Prior to ex div dat	P a> pe	Pa> max (0,PV(D) + XE^{-RT} -SO, X-S

European/call or put – one or more dividend, ex dividend date remaining life of option

$C= P+ S - PV(D) - xe^{-rt}$

American call/put – pay 1 or more ex-div date over remaining life

$P+s - pv(D) -x < C < p+s-xe^{-rt}$