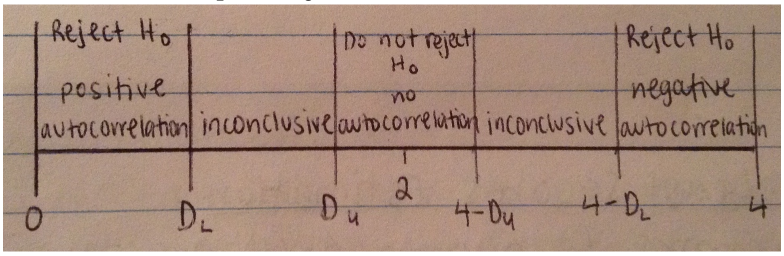
Autocorrelation: MR8: $Cov(e_t\, e_{t-k}) \neq 0,\;\; \sigma_e^2\rho^k$, k > 0 with MR3: $Var(e_t) = \frac{\sigma_v^2}{1-\rho^2} \equiv \sigma_e^2$

- OLS unbiased and consistent, variances no longer valid, OLS inefficient

Detecting Autocorrelation:

1. Residual plots:
   - Positive autocorrelation: residual plots reveal runs of positive residual plots followed by runs of negative residuals.
   - Negative autocorrelation: positive residuals tend to be followed by negative residuals, and negative residuals followed by positive residuals.
2. Residual correlogram:
   Correlation between errors that are k periods apart: $corr(e_t, e_{t-k}) = \frac{cov(e_t, e_{t-k})}{var(e_t)}$
   Correlation = r_k if estimated. Sequence r_1, r_2, … is known as the sample autocorrelation function or correlogram. If $H_0\!: corr(e_t, e_{t-k}) = 0$ is true then $t = r_k\sqrt{T} \approx N(0,1)$ where T is the sample size.
   → can just look at p-value on correlogram
3. Lagrange multiplier test:
   $H_0\!: \rho = 0 \quad H_1\!: \rho \neq 0 \;\; LM = T{\times}R^2 \sim \chi_1^2$ where R^2 is the coefficient of determination in the regression of the $\hat{e}_i$ on a $1, x_{i2}, \dots, x_{ik}\; an\; \hat{e}_{t-1}$. → also known as Breush-Godfrey test.
4. Durbin-Watson test: If the explanatory variables do not include any lagged values of the endogenous variable then we can test $H_0\!: \rho = 0 \quad H_1\!: \rho > 0$ (positive autocorrelation)
   Critical values: d_L and d_U. Obtain Durbin-Watson stat from EViews.



OLS estimation:

- Can still use OLS, but variances are no longer valid
- Heteroskedasticity- and autocorrelation-consistent (HAC) standard errors using an estimator suggested by Newey and West. Newey-West standard errors are analogous to White's standard errors that are used when errors are heteroskedastic.

GLS estimation:

- If errors follow an AR(1) process and ρ is known then we can obtain unbiased and efficient estimates by applying OLS to a transformed model:
  $y_i^* = \beta_1 x_{i1}^* + \beta_2 x_{i2}^* + \dots + \beta_k x_{ik}^* + e_i^*$ where $y_t^* = y_t - \rho y_{t-1},\; x_{t1}^* = 1 - \rho,\;\; x_{tk}^* = x_{tk} - \rho x_{t-1,k}\; for\; k > 1$
  Only T-1 observations are used for estimation (one observation is lost through lagging). Known as the Cochrane-Orcutt transformation. If ρ is unknown, we can use the first-order sample correlation coefficient as an estimator.

Nonlinear Least Squares Estimation:

→ Unbiased and efficient estimates by estimating the model:

$$y_i = \beta_1 x_{i1} + \dots + \beta_k x_{ik} + \rho\big(y_{t-1} - \beta_1 - \beta_2 x_{t-1,2} - \dots - \beta_k x_{t-1,k}\big) + v_t$$

- Nonlinearity make it difficult to find values of the parameters that minimise the sum of squares function.
- Eviews uses nonlinear least squares (NLS). The estimates are found numerically (by systematically evaluating the sum of squares function at different values of the parameters until the least squares estimates are found).
- NLS is equivalent to iterative GLS estimation using the Cochrane-Orcutt transformation.