AERO3360 AEROSPACE STRUCTURES COURSE NOTES

[COMPANY NAME] [Company address]

Contents

AERO3360 AEROSPACE STRUCTURES 1 COURSE NOTES

Lecture 1. Monday, 6 March 2017

Introduction to course

Lecturer: Professor Liyong Tong; liyong.tong@sydney.edu.au

- Monday 2-4 AERO N328 office hour

UOS description:

- Theoretical basis of advanced aerospace structural analysis
- Real-world aircraft structural problems

Attributes:

- An understanding of the derivation of the fundamental equations of elasticity and their application in certain analytical problems;
- An understanding of plate theory and the ability to use this to obtain analytical solutions for plate bending and buckling problems;
- An understanding of energy-method to develop a deeper appreciation for the complexities of designing solution techniques for structural problems;
- An understanding of the basic principals behind stressed-skin aircraft construction and the practical analysis of typical aircraft components, including the limitations of such techniques.

Assessments:

1	Assignment 1	No	5.00	Monday Week 3	4,
2	Assignment 2	No	5.00	Monday Week 6	4,
3	Assignment 3	No	5.00	Monday Week 9	1, 4,
4	Assignment 4	No	5.00	Monday Week 11	1, 2, 3, 4,
5	Assignment 5	No	5.00	Friday Week 13	1, 2, 3, 4,
6	Quiz 1	No	10.00	Week 5 (week 1-5 content)	1, 2, 3, 4,

7	Quiz 2	No	10.00	Week 10 (week 5-10 content)	1, 2, 3, 4,
8	Exam	No	55.00	Exam Period	1, 2, 3, 4,

Assignment: A penalty of 15% per day will be applied for late submission.

Quiz: Each quiz will be closed-book and held during lecture hour.

Exam: This will be a closed-book exam. A student must get 40% in the final exam to pass the unit, regardless of the sum of his/her individual marks.

There may be statistically defensible moderation when combining the marks from each component to ensure consistency of marking between markers, and alignment of final grades with unit outcomes.

Textbook:

Aircraft structures for engineering students, 5th edition, Elsevier 2013

References:

Theory of elasticity; Timoshenko/Goodier

Theory of elastic stability; Timoshenko/Gere

Theory of plates and shells; Timoshenko/Woinowsky-Kreiger

Schedule:

Loads on aircraft (Chap 13-15 in textbook)

2 Function of structural components (Chap 12 in textbook)

3 Bending of beams with non-symmetrical cross-sections (Chap 16 in textbook)

4 Stress, strain and displacement relationships for open and closed single cell thin walled beams

(Chap 17 in textbook)

5 Torsion of thin walled beam sections (Chap 18, 27, 3 in textbook)

6 Structural idealization (Chap 20 in textbook)

7 Multicell beams (Chap 19, 22, 23, 24 in textbook)

8 Tapered beams (Chap 21, 23 in textbook)

9 Shear panels, ribs and cut-outs (Chap 24 in textbook)

10 Structural constraints (Chap 26 in textbook)

11 Elasticity (Chap 1 in textbook)

12 2D problems (Chap 2 in textbook)

13 2D problems in polar coordinates (Chap 4 in R1)

14 Bending of thin plates (Chap 7 in textbook)

15 Plates with combined bending and in-plane loadings (Chap 7, 9 in textbook)

16 Energy methods for plate bending (Chap 7, 5, 4 in textbook)

17 Composite structures (Chap 25 in textbook)

Website:

Username: AERO3360

Password: AS1_Aero1995

Chapter 1: Loads on Aircraft

Types of Loads

An aircraft is required to support two types of basic loads:

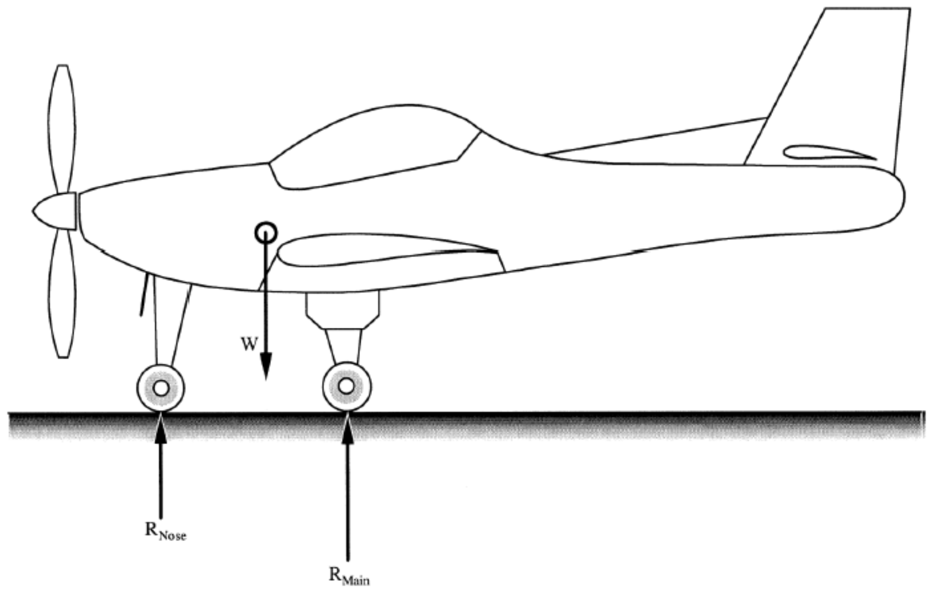
1. Ground loads:
    - Encountered by aircraft during ground movement
        - Taxying
        - Landing
        - Towing
2. Air loads
    - Loads exerted onto the structure during flight by manoeuvres carried out by the aircraft or wind gusts (such as wind shear)

These loads can be further divided into:

- Surface loads
    - Act on the surface of the structure (eg: aerodynamics or hydrostatic loads)
- Body forces:
    - Act over the volume of the structure and are generated by gravitational and inertial effects

Ground Loads:

- Loads experienced include weight and reactions

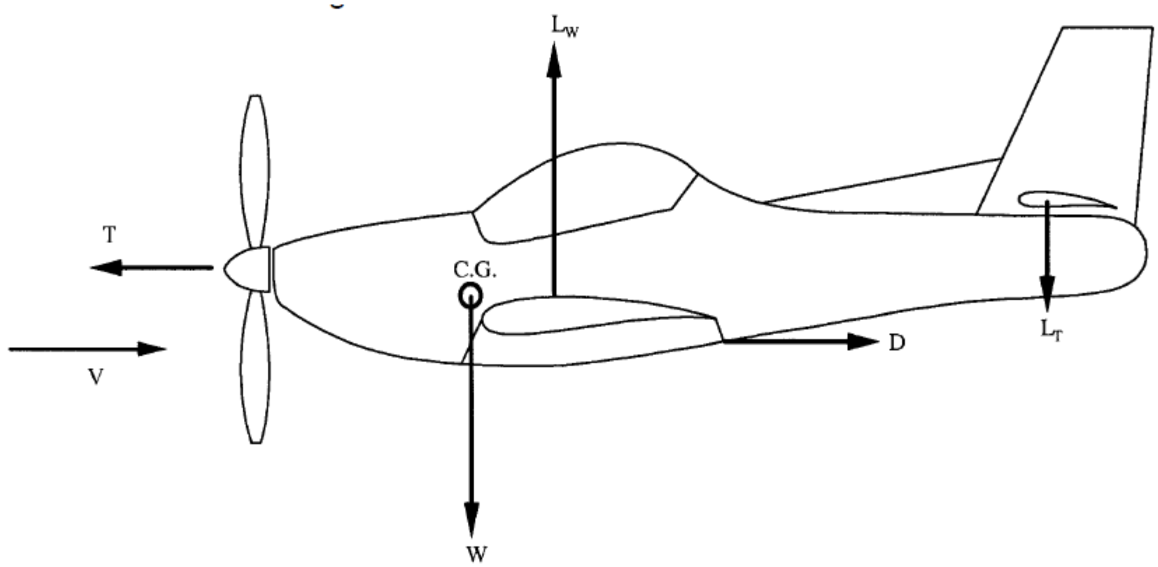


Air loads

Loads experienced can change depending on manoeuvres

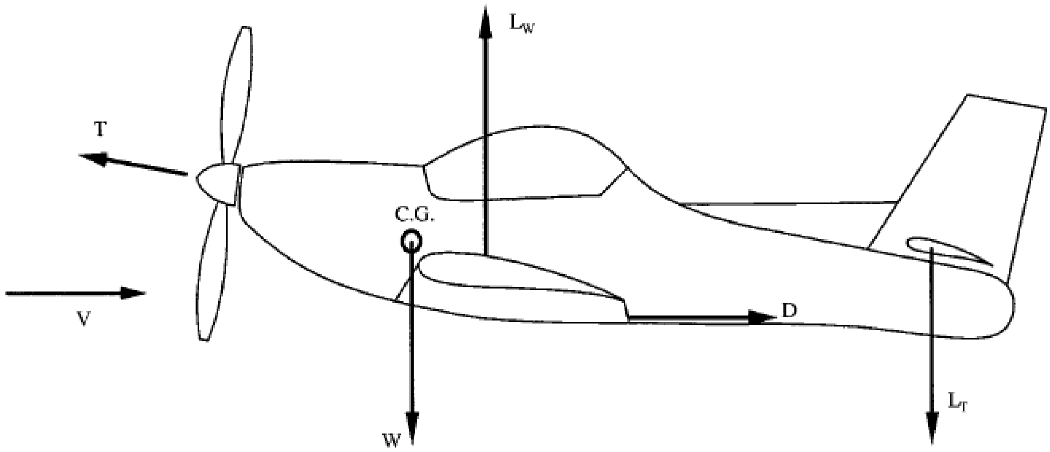
Level flight

- Thrust, lift, weight, drag



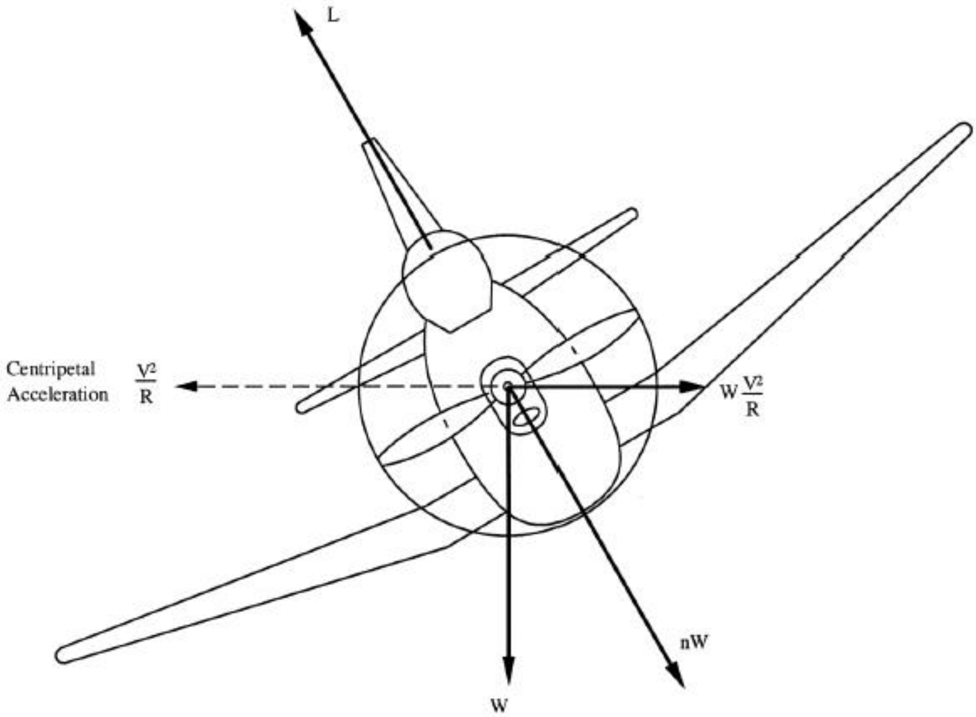
Pull up flight:

- Orientation of some forces changes when pulling up



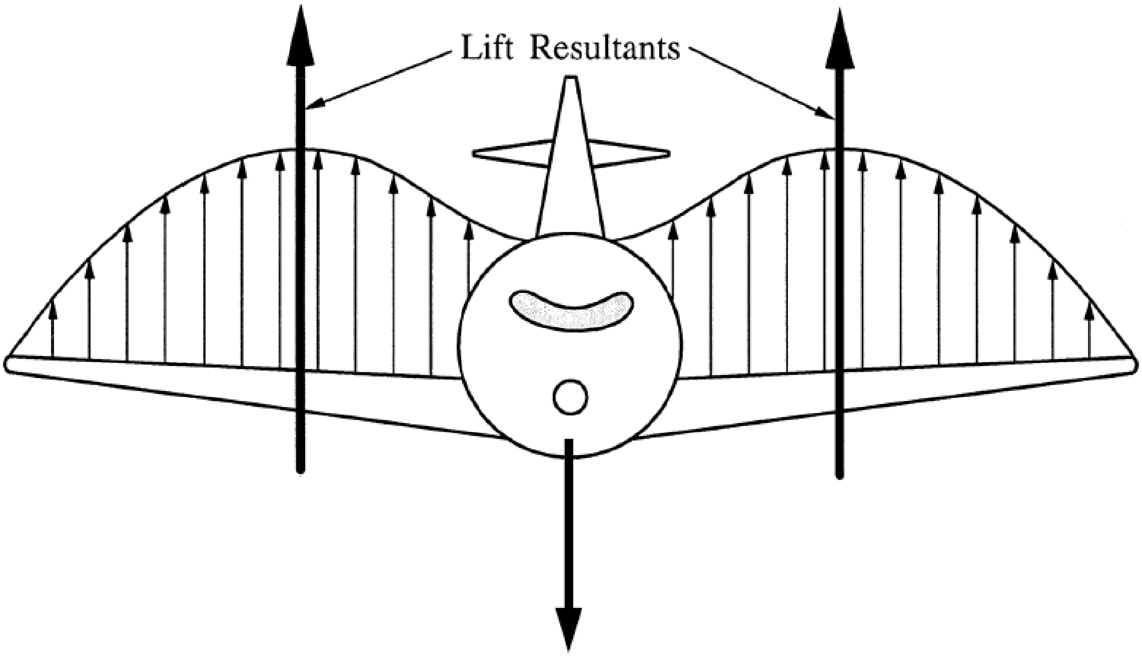
Steady banked turn

- Lift vector changes towards angle of turn.



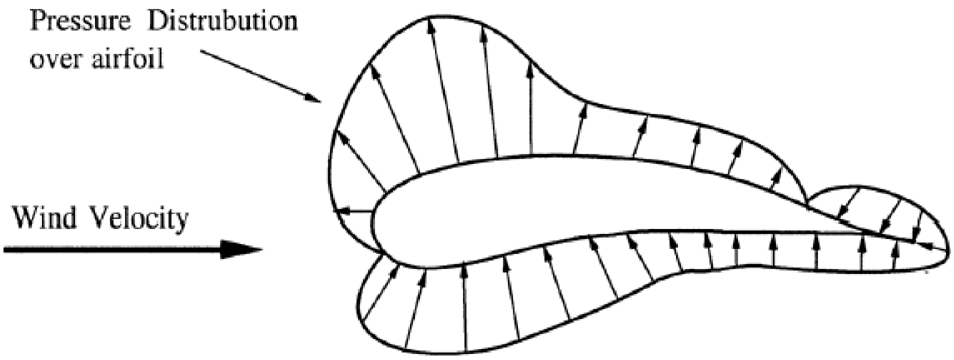
Aerodynamic surface loads

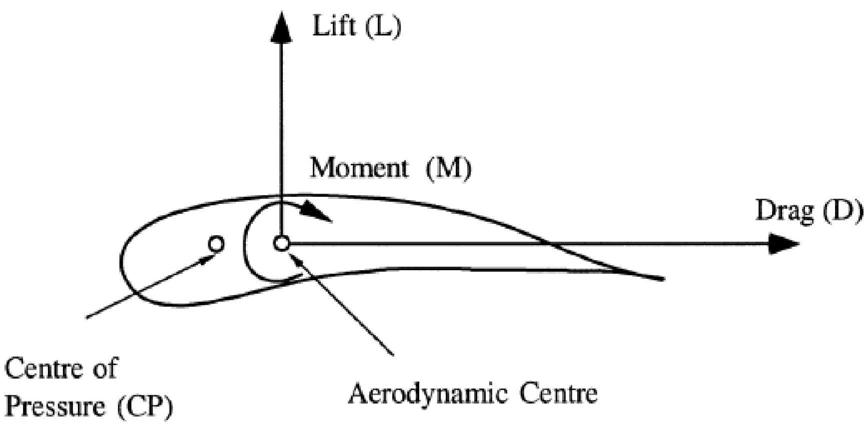
Spanwise



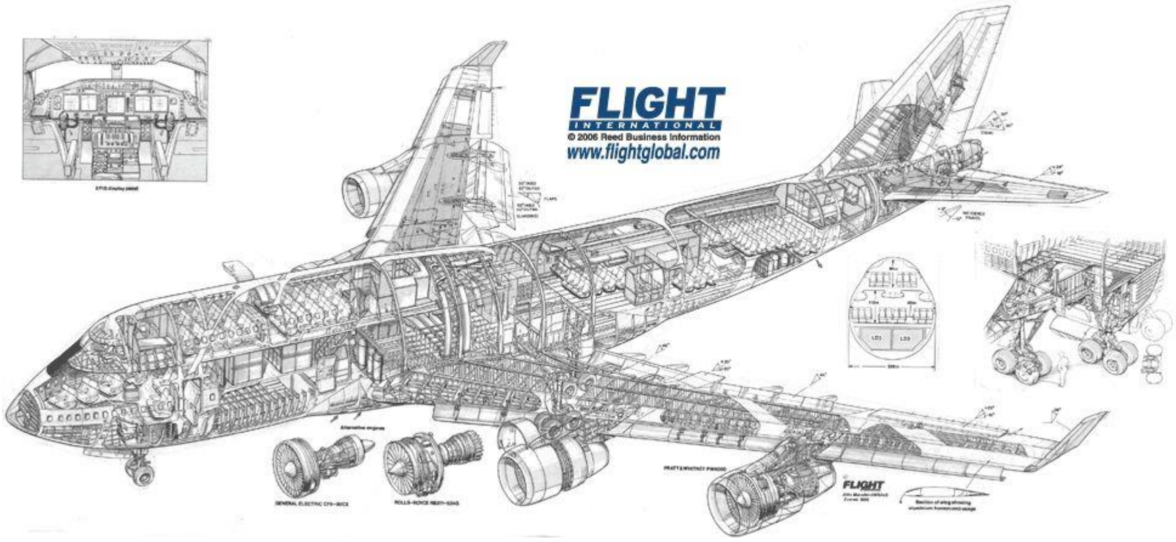
- Bending moments, shear, torsion, tension/compression ect

Chordwise



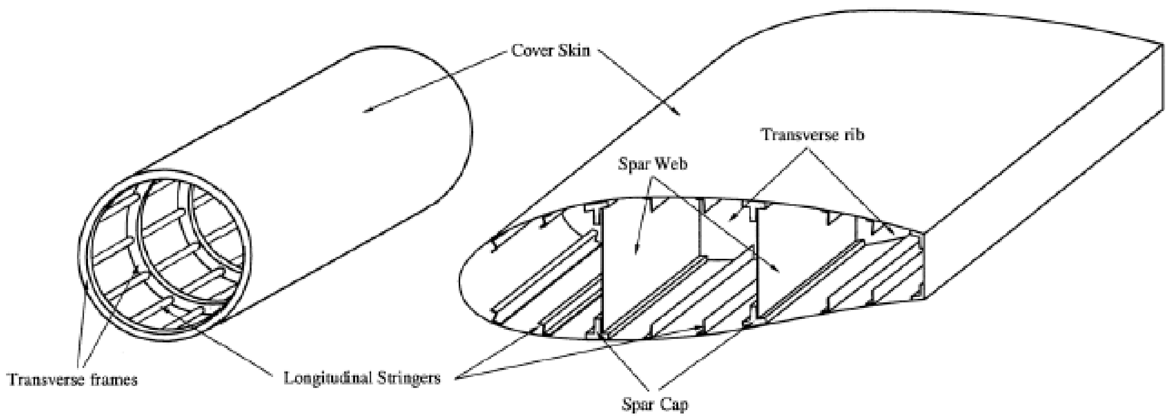


Chapter 2: Function of Structural Components

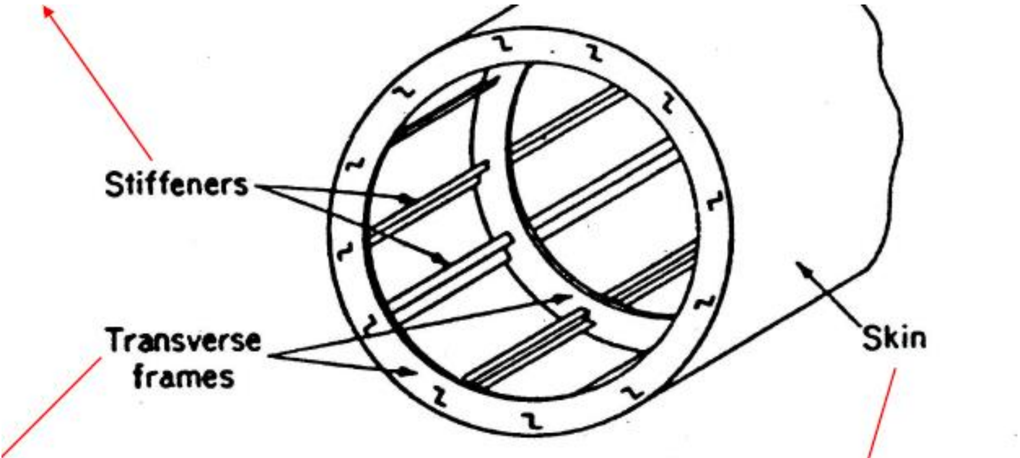


Typical semi-monocoque structural components

- Cover skin
- Spar web
- Transverse rib
- Transverse trames
- Spar cap
- Longitudinal stringers



Stiffeners:



Arrangement of skin, longitudinal stiffeners and transverse frames in a typical fuselage section.

- Resist bending and axial loads
- Divide the skin into small panels
- Act with skin to resist axial loads caused by pressurisation

Skin:

- Transmits aerodynamics forces to the longitudinal and transverse members
- Develops shear stresses to react the applied torsional moments
- Together with longitudinal members, it resists axial loads and applied bending

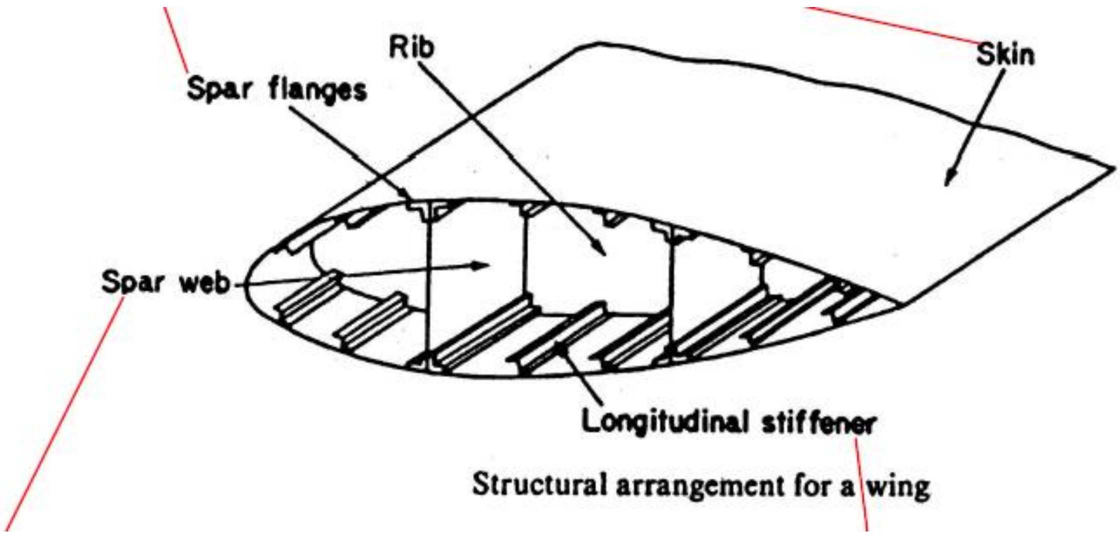
Transverse frames

- Maintains cross sectional shape
- Distributed concentrated loads
- Establishes column length to prevent buckling
- Provides edge restraint for skin
- Acts with skin to resits circumferential loading

Typical semi-monocoque construction of aircraft wing

Spar flanges

- Resist bending and axial loads
- Divide skin into small panels



Structural arrangement for a wing

Skin:

- Transmits aerodynamics forces to the longitudinal and transverse members
- Develops shear stresses to react the applied torsional moments
- Together with longitudinal members, it resists axial loads and applied bending

Spare web

- Develops shear stresses to react the applied torsional moments

Longitudinal stiffeners

- Resists bending and axial loads
- Divides skin into small panels
- Acts with skin to resist axial loads caused by pressurisation

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Lecture 2. Tuesday, 7 March 2017

Chapter 3: Bending of Beams with non-symmetrical cross section

Sign convention and notation:

Forces and moments:

$T =$ torque.

- Positive using right hand rule with thumb out of beam

$M =$ Bending moment.

- M_x and M_y are positive if they induce a positive stress in the positive xy quadrant of the beam.

$S =$ Shear force.

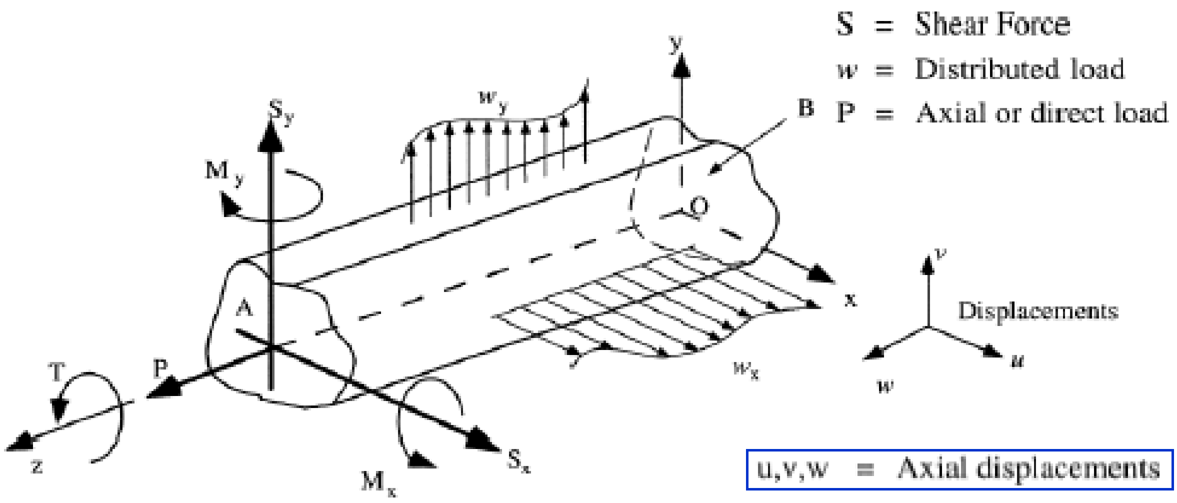
- Positive in positive x, y, z depending on orientation

$w =$ distributed load.

- Positive out

$P =$ axial/direct load.

- Positive out of beam



However on face B, the positive internal moments and forces are in the opposite sense.

0 moments M_x and M_y are positive if they induce a positive stress in the positive xy quadrant

Displacements:

u, v, w are x, y, z displacements in axial direction

Placement of origin:

- MUST adhere to right hand rule.

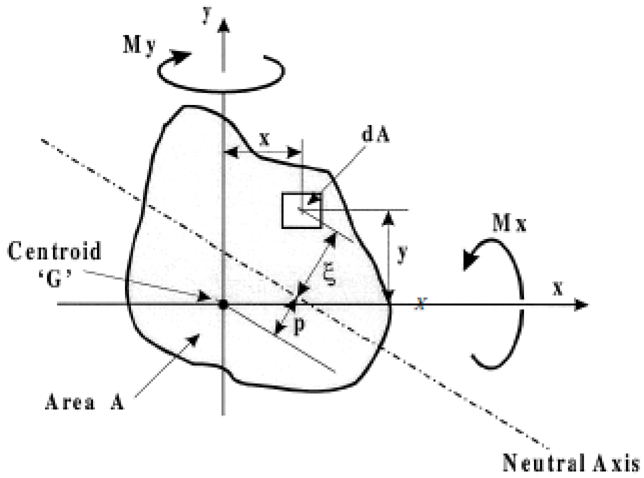
Bending of a beam

Recall: bending of beam with symmetrical cross section

Recall from Mech of solids 1: the bending moment on a symmetrical cross section is $\sigma = \frac{My}{I}$; eg a circular, I beam or rectangular cross section.

Non symmetric cross section:

Step 1: Determination of centroid and neutral axis



$$\bar{x} = \frac{\sum \bar{x}_i A_i}{\sum A_i}$$

Note:

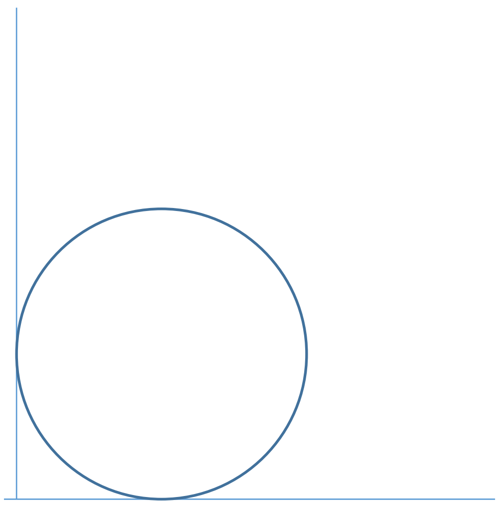
Centroid

- the centre of mass of a geometric object of uniform density.

Neutral axis

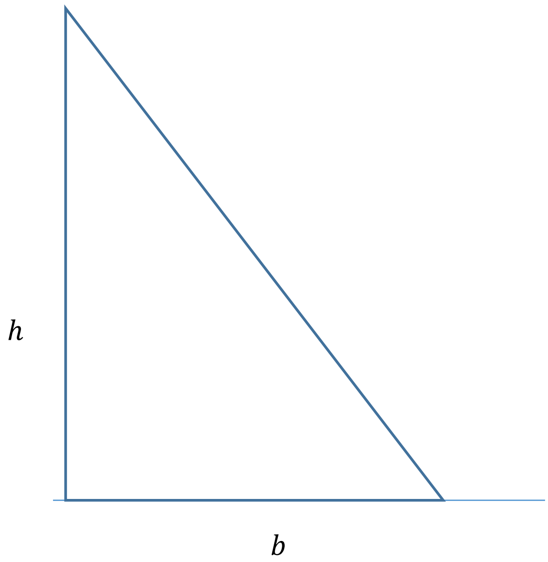
- a line or plane through a beam or plate connecting points at which no extension or compression occurs when it is bent.
- No bending stress

Example: finding centroid



$$\bar{x} = \frac{\int_A x dA}{\int_A dA}$$
$$\bar{y} = \frac{\int_A y dA}{\int_A dA}$$

For a triangle:



$$\therefore \bar{x} = \frac{\int_0^h \int_0^x x dx dy}{\frac{1}{2} bh}$$
$$= \frac{b}{3}$$

Parallel axis theorem:

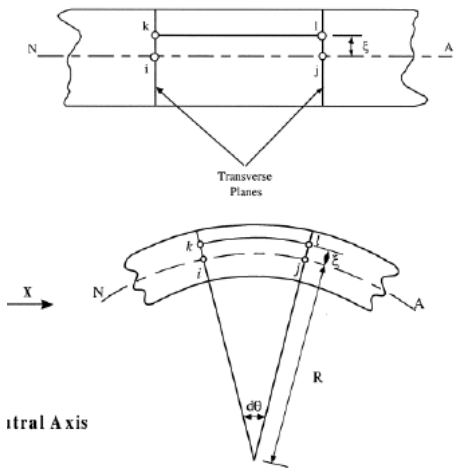
$$I'_{xx} = \sum_i \left(I_{xx_i} + y_i^2 A_i \right)$$
$$I'_{yy} = \sum_i \left(I_{yy_i} + x_i^2 A_i \right)$$
$$I'_{xy} = \sum_i \left(I_{xy} + x_i y_i A_i \right)$$

Where I'_{xx} is the moment of area about the centroid, I_{xx} is the moment of area of a shape, y_i is the distance from centroid of shape to total centroid, A is area of the shape.

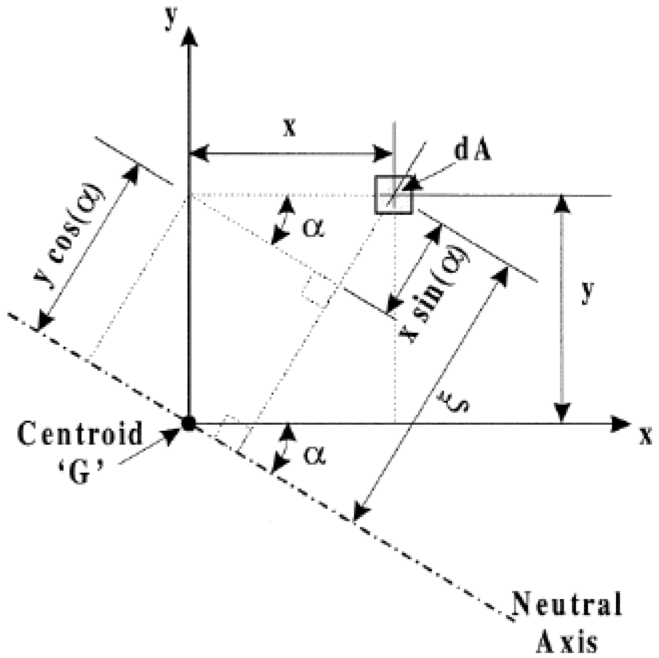
Note: no geometric property equation provided in exam.

Lecture 3. Monday, 13 March 2017

$$\sigma_z = \frac{\xi E}{R}$$



$$\epsilon = \frac{\Delta L}{L} = \frac{kl - ij}{ij}$$



Stress at point (x, y) or dA

$$\sigma_x = \frac{E}{R}(x \sin\alpha + y \cos\alpha)$$

Moment equilibrium:

$$M_x = \int_A \sigma_z y dA$$

Perpendicular distance of dA from NA

$$\xi = x \sin\alpha + y \cos\alpha$$

Normal bending stress:

$$\boxed{\sigma_z = \left(\frac{M_y I_{xx} - M_x I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right) x + \left(\frac{M_x I_{yy} - M_y I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right) y}$$

Or:

$$\sigma_{x_i} = \frac{\overline{M_x}}{I_{xx}} y_i + \frac{\bar{M}_y}{I_{yy}} x \pm \frac{P_z}{A}$$

---

if

$$\bar{M}_x = \frac{M_x - M_y \frac{I_{xy}}{I_{yy}}}{1 - \frac{I_{xy}^2}{I_{xx} I_{yy}}}$$

$$\bar{M}_y = \frac{M_y - M_x \frac{I_{xy}}{I_{xx}}}{1 - \frac{I_{xy}^2}{I_{xx} I_{yy}}}$$

(note $P_z = 0$ if no applied load)

Where:

$$I_{xx} = \int_a y^2 dA$$

$$I_{xy} = \int_A xy dA$$

(note: for symmetric shapes, $I_{xy} = 0$)

Inclination angle of neutral axis:

The NA is when $\sigma_z = 0$

Solving this using or formula for σ_z gives

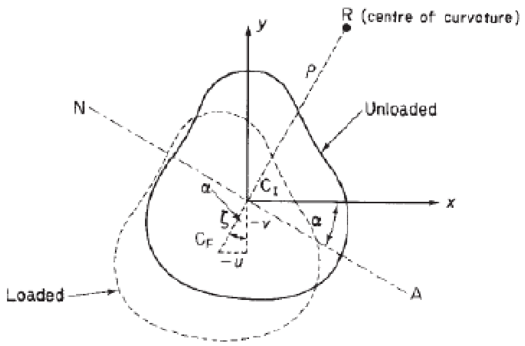
$$\boxed{\tan \alpha = -\frac{y_{NA}}{x_{NA}} = \frac{M_y I_{xx} - M_x I_{xy}}{M_x I_{yy} - M_y I_{xy}}}$$

Deflection due to bending:

When a beam bends about its neutral axis, the deflection normal to the NA ζ, can be used to define the curvature ratio:

$$\frac{1}{\rho} = \frac{d^2 \zeta}{dz^2}$$

- And similarly in the x, y direction of deflection



Differentiating both twice wrt z:

$$\frac{\sin\alpha}{\rho} = -\frac{d^2u}{dz^2}$$
$$\frac{\cos\alpha}{\rho} = -\frac{d^2v}{dz^2}$$

Substituting in $\sigma_z = \frac{E\xi}{\rho} = \frac{E}{\rho}(x\sin\alpha + y\cos\alpha)$ and the moments:

$$M_x = -E\left(I_{xy}u'' + I_{xx}v''\right)$$
$$M_y = -E\left(I_{yy}u'' + I_{xy}v''\right)$$

Moment curvature relation:

This can be expressed as a matrix:

$$\boxed{\begin{pmatrix} M_x \\ M_y \end{pmatrix} = -E\begin{pmatrix} I_{xy} & I_{xx} \\ I_{yy} & I_{xy} \end{pmatrix}\begin{pmatrix} u'' \\ v'' \end{pmatrix}}$$

Summary: Bending of with non-symmetrical cross section

Bending stress:

$$\sigma_z = \left(\frac{M_yI_{xx} - M_xI_{xy}}{I_{xx}I_{yy} - I_{xy}^2}\right)x + \left(\frac{M_xI_{yy} - M_yI_{xy}}{I_{xx}I_{yy} - I_{xy}^2}\right)y \pm \frac{P_z}{A}$$

Where: $I_{xx} = \iint_A y^2 dA\,;I_{yy} = \iint_A x^2 dA\,;I_{xy} = \iint_A xy dA$

Direction of Neutral Axis:

$$\tan\alpha = \frac{y_{NA}}{x_{NA}} = \frac{M_yI_{xx} - M_xI_{xy}}{M_xI_{yy} - M_yI_{xy}}$$

(take downwards as positive)

Moment- Curvature relation

$$\begin{pmatrix} M_x \\ M_y \end{pmatrix} = -E\begin{pmatrix} I_{xy} & I_{xx} \\ I_{yy} & I_{xy} \end{pmatrix}\begin{pmatrix} u'' \\ v'' \end{pmatrix}$$

Solution steps:

1. Determine location of centroid about a selected origin
2. Shift the axis to the centroid
3. Determine sectional properties (I_{xx}, I_{yy}, I_{xy} and A of each segment)
4. Calculate bending stress

Example 1:

Example 1: The beam shown is subjected to a bending moment of 150Nm about the x-axis. Calculate the maximum direct stress due to bending stating where it acts.

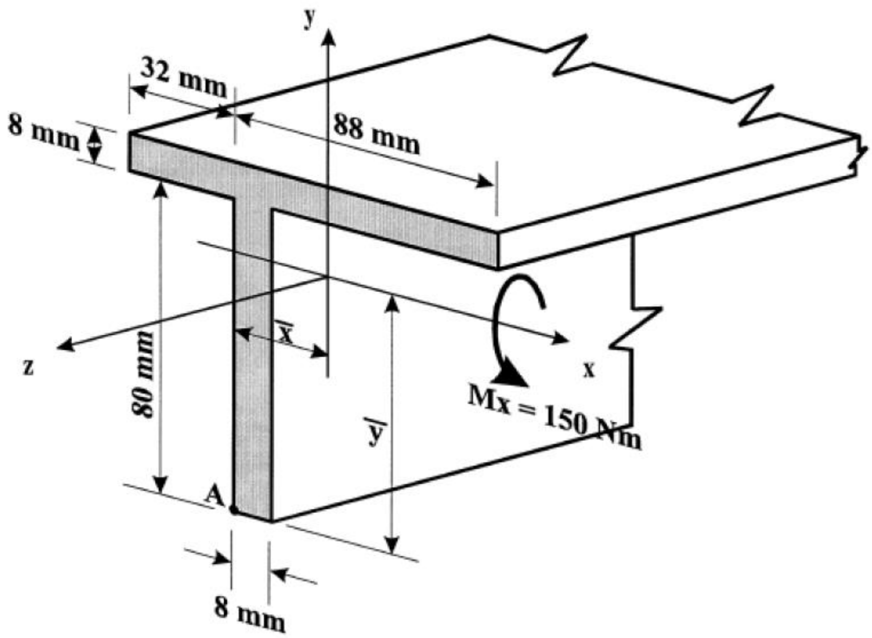


Figure 15: Beam cross section with applied bending moment

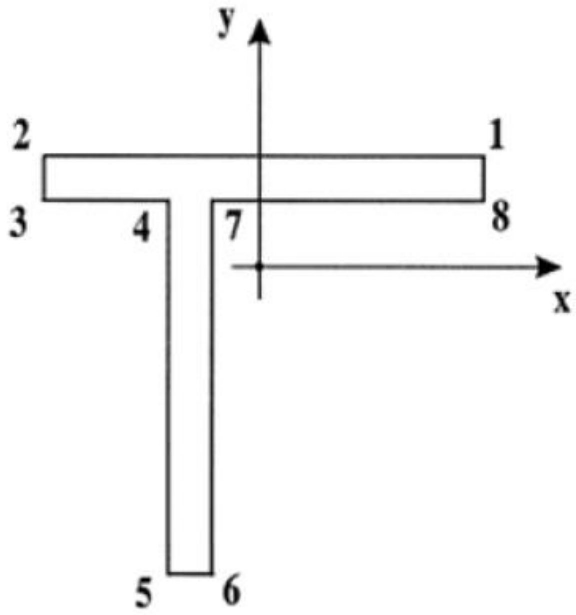
Determine location of centroid: (take A as origin)

$$\bar{x} = \frac{\sum \bar{x}_i A_i}{\sum A_i} = \frac{120(8)(28) + 80(8)(4)}{120(8) + 80(8)} = 18.4mm$$

Similary:

$$\bar{y} = 66.4mm$$

Shift axis to centroid:



Determine sectional properties:

$$I_{xx} = \sum I_{xx} + y^2A = \frac{120 \times 8^3}{12} + 120 \times 8 \times 17.6^2 + 8 \times \frac{80^3}{12} + 80(8)(26.4)^2 = 1.09 \times 10^6 mm^4$$

Similarly:

$$I_{yy} = 1.38 \times 10^6$$
$$I_{xy} = 0.405 \times 10^6$$

Finding bending stress:

$$\sigma_z = \left(\frac{M_y I_{xx} - M_x I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right) x + \left(\frac{M_x I_{yy} - M_y I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right) y$$

Stress at each point: draw up a table with each xy coordinate of each point

Point	x	y	sigma-z
1	69.6	21.6	0.15
2	-50.4	21.6	5.02
3	-50.4	13.6	3.92
4	-18.4	13.6	2.62
5	-18.4	-66.4	-8.39
6	-10.4	-66.4	-8.72
7	-10.4	13.6	2.29
8	69.6	13.6	-0.95

Approximation for thin walled sections:

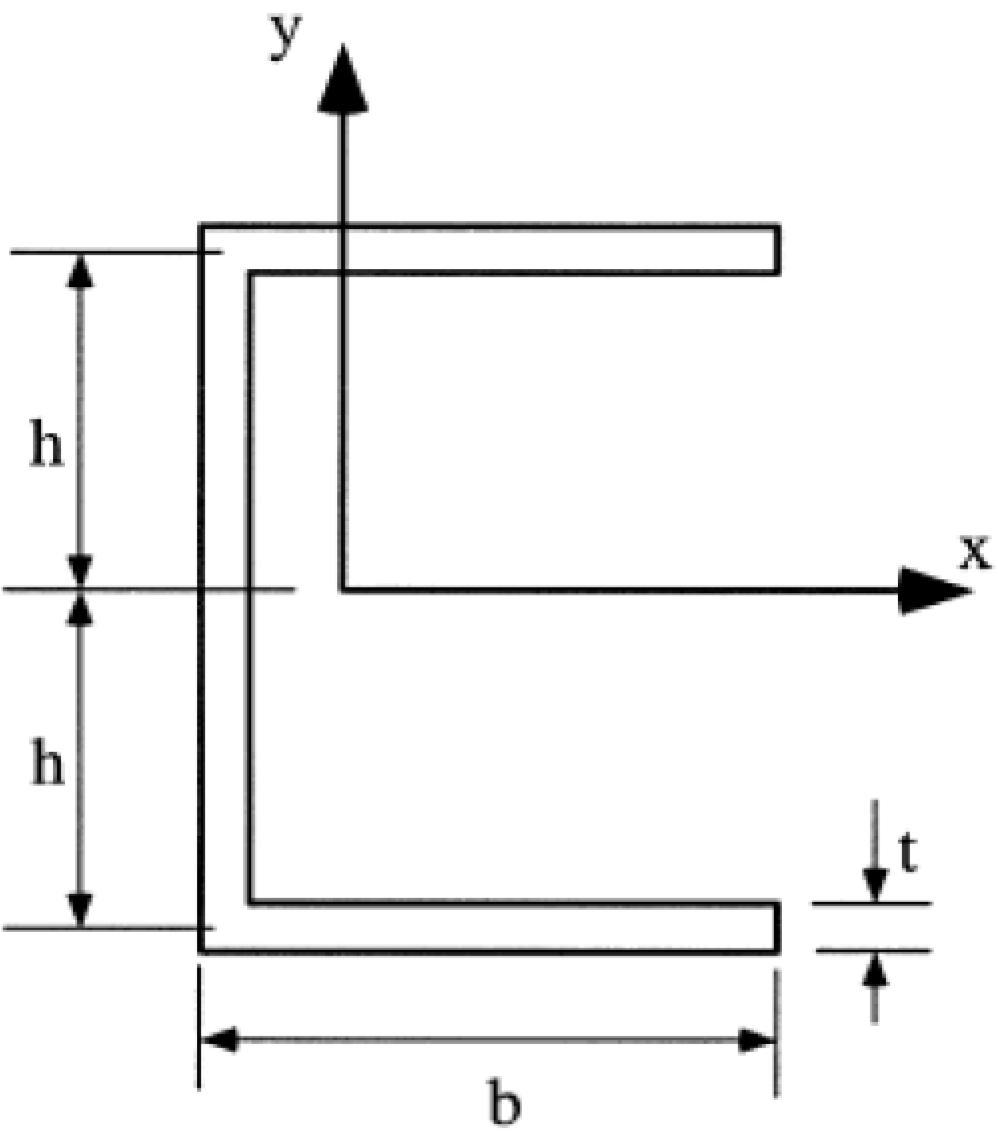


Figure 17: Thin-walled channel section

As $t \ll h$, can approximate $t^3 \approx 0$

Due to symmetry:

$$
\begin{aligned}
I_{xy} &= 0 \\
I_{xx} &= 2bt^2 + 2bth^3 \\
I_{yy} &= \frac{2tb^3}{12} + 2bt\left(\frac{hb}{2b + 2h}\right) + 2ht\left(\frac{b^2}{2b + 2h}\right)
\end{aligned}
$$

With

$$\bar{x} = \frac{b^2}{2b + 2h}$$
$$\bar{y} = h$$

Second moment of area for thin walled approximations:

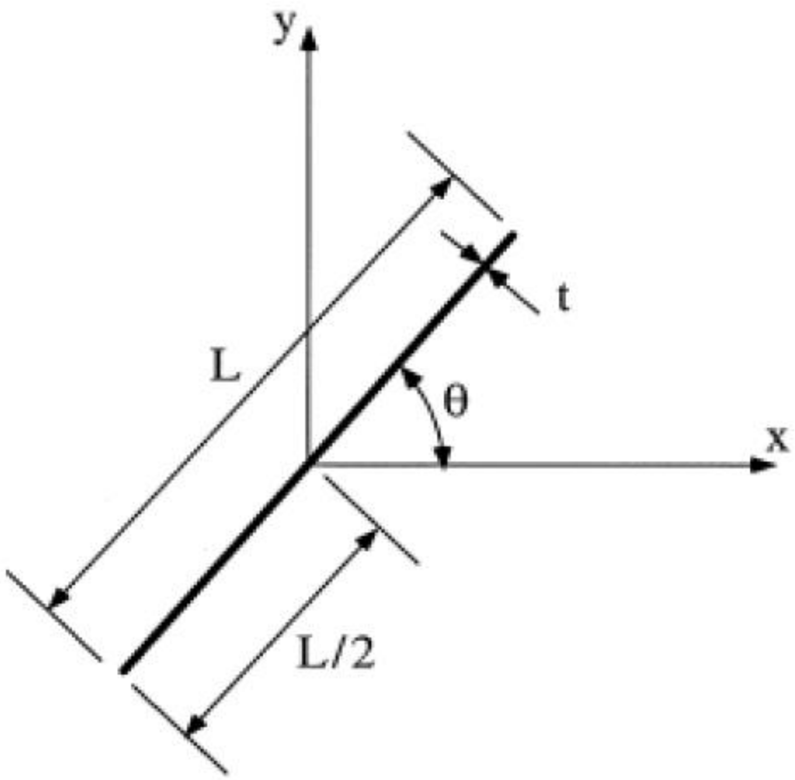


Figure 20: Thin skin section inclined at an angle θ wrt the x-axis

$$I_{xx} = \frac{L^3 t \sin^2 \theta}{12}$$
$$I_{yy} = \frac{L^3 t \cos^2 \theta}{12}$$
$$I_{xy} = \frac{L^3 t \sin 2\theta}{24}$$

General loading relationships:

- Cutting a loaded beam for a small section δz: