Contents

Potential flow formulas................................................................. 7
Introduction ................................................................................... 8
  What will we study?.................................................................. 8
  Challenges: ............................................................................. 8
  Assignments: .......................................................................... 8
  Resources: ............................................................................... 8
  Current assignment: ................................................................. 9
Aerofoil features .......................................................................... 9
Air mach numbers ......................................................................... 9
Introduction: ................................................................................ 9
  Wing section: .......................................................................... 9
  Why Aerofoils work.................................................................. 10
  Wing section types: ................................................................. 11
  Aerofoil with camber ............................................................... 13
  NACA Aerofoil ...................................................................... 15
  Effects of wing section: ........................................................... 16
Writing reports ............................................................................. 17
Lab report Components: ............................................................... 17
  Structure ................................................................................. 17
  Components: .......................................................................... 17
Aerofoils (again) .......................................................................... 19
Stall ............................................................................................ 19
  Factors that contribute to stall ................................................. 20
  Reducing stall abruptness: ...................................................... 21
  Leading edge stall: ................................................................. 21
  Trailing edge stall: ................................................................. 22
Selecting an aerofoil .................................................................... 23
Maximising lift ............................................................................ 24
Increasing lift .............................................................................. 24
  High lift devices ................................................................... 25
  Supercritical aerofoils ............................................................ 28
Navier Stokes and Potential Flow theory ..................................... 32
Equations of motion: Navier stokes equation ............................ 32
  Derivation: ............................................................................ 32
  Continuity equation ............................................................... 33
Navier stokes equation ................................................................. 33
Potential flow .................................................................................. 34
Point vortex ..................................................................................... 43
Complex potential ............................................................................ 43
Stream: ............................................................................................ 43
Source .............................................................................................. 43
Vortex ............................................................................................... 44

2D aerofoil theory .......................................................................... 62
Introduction ...................................................................................... 62
Modelling wing section ................................................................... 62
Conformal mapping .......................................................................... 62
Joukowskï transformation ............................................................... 63
Lifting flat plate (transforming spinning circular cylinder) .......... 65
Vortices (tute) .................................................................................. 70
Thick aerofoils .................................................................................. 71
Leading and trailing edge ................................................................. 71
Circulation around thick aerofoils .................................................. 72
Introducing camber: ......................................................................... 73
Karman – treffits aerofoil ............................................................... 77
Generalised Karman Treffitz conformal map .................................. 77
Theodorsen aerofoil design ............................................................. 80
The $\zeta$ plane: (aerofoil) ................................................................. 81
Theodorsen aerofoil design ............................................................. 82
Thin aerofoil theory ......................................................................... 82
Camber line ...................................................................................... 82
Uncambered aerofoil ....................................................................... 84
Cambered aerofoils .......................................................................... 86
Comparison of 2D aerofoil theories: .............................................. 89
Tute: thin aerofoil theory ................................................................. 89

2D panel method ............................................................................ 90
Panel placement ............................................................................... 91
Problem statement: ........................................................................ 91
Higher order accuracy than panel method: .................................. 101
Viscous – inviscid interaction: ....................................................... 103
EIF construction ............................................................................. 103
Solving: Method 1: .......................................................................... 104
Solving: Method 2: ................................................................. 104

**Design of aerofoils:** ........................................................................................................... 104

**Optimisation techniques:** ............................................................................................... 104
- Optimisation theory: ........................................................................................................... 104
- Evolutionary algorithms: .................................................................................................... 105
- Some objective functions: .................................................................................................. 108
- PARSEC Method: ................................................................................................................ 110
- Inverse design methods: .................................................................................................... 111
- Potential flow in 3D: ........................................................................................................... 115

**Wind tunnel experiments and measurements** ................................................................. 117
- Introduction: ....................................................................................................................... 117
- Types of aerodynamics experiments: .................................................................................. 118
- Wind tunnel principles: ...................................................................................................... 120
- Scale parameters: ................................................................................................................ 120
- Open circuit wind tunnels: .................................................................................................. 121
- Closed circuit wind tunnel ................................................................................................ 121
  - Advantages/disadvantages: ............................................................................................... 121
  - Wind tunnel dimensions: .................................................................................................. 123
  - Test section: ..................................................................................................................... 124
- Vanes: .................................................................................................................................. 126
  - Corner loss coefficient: ...................................................................................................... 126
  - Vane design: ..................................................................................................................... 127
- Fans: .................................................................................................................................... 127
  - Purpose of the Fan: ............................................................................................................ 127
  - Fan section: ..................................................................................................................... 128
- Honeycomb: ....................................................................................................................... 128
- Screens: ............................................................................................................................... 129
- Contraction cone: ............................................................................................................... 129
  - Contraction design: ......................................................................................................... 129
- Cooling of wind tunnels: ..................................................................................................... 129
- Cooling methods: ............................................................................................................... 129
- Flow quality: ....................................................................................................................... 130
- Steadiness: .......................................................................................................................... 130
- Turbulence: ......................................................................................................................... 130
- Special kinds of tunnels: ..................................................................................................... 130
- Transonic tunnels: .............................................................................................................. 130
Supersonic tunnels: ................................................................. 131
Hypersonic tunnels: ............................................................. 132
Icing tunnels: ........................................................................ 132
Automotive tunnels: ............................................................. 133
Water tunnels: ....................................................................... 133
Energy of a tunnel: .................................................................. 134
Energy ratio ........................................................................... 134
Loss around tunnel: ................................................................ 134
Close circuit tunnel losses (generally)................................... 135
Instrumentation testing and procedure: ............................... 135
Introduction: .......................................................................... 135
Working section airspeed: .................................................... 136
Instruments: ........................................................................... 136
Flow visualisation: ................................................................... 141
Aerodynamic load measurements: ....................................... 151
Pressure distribution measurements: ................................... 152
Experimental error: ............................................................... 154
Common sense errors: ......................................................... 154
Wind tunnel correction: ....................................................... 155
3D FLOW ................................................................................ 168
End plates ............................................................................. 168
3D flow: ................................................................................ 168
3D flow basics: ..................................................................... 169
Flow streamlines: ................................................................... 169
Wingtip vortices: .................................................................... 171
3D aerodynamic feedback loop: .......................................... 172
Modelling of flow around 3D wings ..................................... 173
Vortex filament and tubes: ................................................... 174
Simplified Lift distribution: ................................................... 176
Horseshoe vortex: ............................................................... 176
3D flow field: ....................................................................... 180
Elliptical lift distribution: ..................................................... 182
Induced drag: ....................................................................... 184
General loading equation .................................................... 185
Aerodynamics of General wing planforms: .......................... 187
Vortex Lattice method: ....................................................... 188
Thwaites Method: ................................................................. 256
For aerofoils: ................................................................. 257
Blasius wall shear stress ....................................................... 257
Potential flow formulas

<table>
<thead>
<tr>
<th>name</th>
<th>velocity</th>
<th>Stream function</th>
<th>Potential function</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uniform (Cartesian)</td>
<td>$(U, 0, 0)$</td>
<td>$\psi = Uy$</td>
<td>$\phi = Ux$</td>
</tr>
<tr>
<td>Uniform (cylindrical)</td>
<td>$u = (0, 0, U)$</td>
<td>$\psi = \frac{1}{2} UR^2$</td>
<td>$\phi = Ux$</td>
</tr>
<tr>
<td>Simple source (spherical)</td>
<td>$(\frac{S}{4\pi r^2}, 0, 0)$</td>
<td>$\psi = \frac{S}{4\pi} \cos \theta$</td>
<td>$\phi = -\frac{S}{4\pi}$</td>
</tr>
<tr>
<td>Dipole source</td>
<td>$M \left( \frac{2 \cos \theta \sin \theta}{r^3}, r^3, 0 \right)$</td>
<td>$\psi = \frac{M \sin^2 \theta}{4\pi r}$</td>
<td>$\phi = -\frac{M}{4\pi r^2}$</td>
</tr>
<tr>
<td>Line source</td>
<td>$u_r \hat{r}$</td>
<td>$S \theta$</td>
<td>$\frac{S}{2\pi} \ln r$</td>
</tr>
<tr>
<td>Line dipole</td>
<td>$(\frac{eS}{4\pi r^2} \cos \theta, \frac{eS}{4\pi r^2} \sin \theta, 0)$</td>
<td>$\frac{eS}{4\pi r} \sin \theta$</td>
<td>$-\frac{eS}{4\pi r} \cos \theta$</td>
</tr>
<tr>
<td>Stagnation point</td>
<td>$(Ax, -Ay, 0)$</td>
<td>$Ax y$</td>
<td>$\frac{1}{2} A(x^2 - y^2)$</td>
</tr>
<tr>
<td>Axisymmetric stagnation point</td>
<td>$(\frac{1}{2} Ar, 0, -Az)$</td>
<td>$-\frac{1}{2} AR^2 z$</td>
<td>$\frac{1}{4} A(R^2 - 2z^2)$</td>
</tr>
<tr>
<td>Line vortex (polar)</td>
<td>$(0, \frac{\Gamma}{2\pi r})$</td>
<td>$-\frac{1}{2} \ln r$</td>
<td>$\frac{\Gamma}{2\pi}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Cartesian $(x, y)$</th>
<th></th>
<th>Polar $(r, \theta)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u = \frac{\partial \phi}{\partial x}$</td>
<td>$u = \frac{\partial \psi}{\partial r}$</td>
<td></td>
</tr>
<tr>
<td>$v = \frac{\partial \phi}{\partial y}$</td>
<td>$v = \frac{\partial \psi}{\partial \theta}$</td>
<td></td>
</tr>
<tr>
<td>$r \frac{\partial \phi}{\partial \theta}$</td>
<td>$\frac{1}{r} \frac{\partial \psi}{\partial r}$</td>
<td></td>
</tr>
</tbody>
</table>

---

Gareth Vlo

Lecture 1. Tuesday, 26 July 2016
Introduction
- Viscosity
  o Stall
  o Dynamic stall
  o Separation
  o Laminar/turbulent flow
  o Pressure gradients

What will we study?
- Potential flow theory (incompressible)
  o 2D building blocks; learning to analyse aerofoil properties
  o Its extension to 3D
- Wind tunnel theory
- Boundary layer theory
  o Estimate viscous drag
- Aerofoil behaviour
  o Geometric properties
- Compressibility effects
- Intro to CFD
  o Terminology. Solutions. Pitfalls, advantages

Challenges:
- Incompressible aerodynamics is lots of maths
- What we see mathematically is very simplified; and lost physical reality.

Assignments:
3 assignments (5, 10, 10) released weeks 1, 5, 10 (last is group)

3 labs (5, 10, 5)
- Week 2/3 cylinder flow
- Week 5/6 2D aerofoil flow
- Week 10/11 3D flow
  o Reports due one week after lab session

Weekly submission of exercise (5)

Exam (50%) (need 40% in exam)

Resources:
- Anderson; fundamentals of aerodynamics
- Abbott: theory of wing sections
- Kuetehe; foundations of aerodynamics
- Bertin; aerodynamics for engineers
- Doug’s aerodynamics for students
Current assignment:
- Page limit
- 2 parts
  - Part A: requires you to solve a problem (which aerofoil to use)
- Part B
  - Research and literature survey
- Submission 2PM Thursday week 4
- Intermediate submission with penalty applied (week 2 and 3)

Lecture 2. Thursday, 28 July 2016

Assignment: just assume unitary span; only have lift and weight forces

\[
\frac{L}{\text{unit span}} = \frac{1}{2} \rho U^2 (\text{chord}) C_L
\]

---

**Aerofoil features**

**Air mach numbers**

- Subsonic \(M < 1\)
- Supersonic \(M > 1\)
- Sonic \(M = 1\)
- Hypersonic \(M > 5\)
- Transonic \(0.75 < M < 1.2\)

**Introduction:**
- Most important phenomenon in a wind tunnel studied is flow around wing
- Several major improvements in aircraft aerodynamics have resulted from the study of new forms of wing sections
  - (make sure you know temp and pressure for wind tunnel experiments)

**Wing section:**
Wing section is a 2D cut out of a wing.

- Its shape is crucial to the aerodynamic performance of the wing
  - Lift curve; \( \frac{dc}{d\alpha} = 2\pi \) (need to remember this idealisation)
  - Drag curve
  - Moment curve; \( \frac{dc}{d\alpha} = 1.8\pi \left( 1 + 0.8 \frac{c_{\text{max}}}{c} \right) \)
\( \alpha \) is the angle of attack

2D flow uses lower case \( c_l, c_d \); 3D flow upper case \( C_L, C_D \)

**Why Aerofoils work**

Bernoulli: longer travel time on top surface (incorrect)

Newton: air hitting lower surface of wing and reaction force; assuming that \( \alpha \) is important to

Newton:

1. In order for the aerofoil to go up, air must go down (true)
2. So angle of attack is important (top plays no part) (half true)

Bernoulli:

1. Air is redirected up on top of wing (true)
2. Air travels further on top (true; not relevant)
3. Air must come back to meet bottom surface at same time (FALSE)
4. Therefore air on top is faster (true, not for given reason)
5. By Bernoulli’s equation, faster air on top exerts less pressure (true, not for given reason)

**Reality:**

Need both to explain

Air is forced down (downwash) due to a speeding up of air on the top of the wing; (Bernoulli effect phenomenon)

The effect is so pronounced that air passing over the wing passes around wing faster that those on bottom.

- Calculation shows that positive pressure on the bottom of wing is insufficient for the entire explanation
- It is negative pressure on top of the wing which accounts for most lift

Downwash is strongest near body of plane and weakest at wingtips. Effect produces vortices in the downwash.
**Boundary layer**

Most of flow field does not be effected by wing;

Only thin boundary layer; if we assume boundary layer is not there we can get a reasonable approximation of lift, but drag is non existent (as no viscosity)

\[ C_D = C_{D0} + kC_L^2 \]

**Wing section types:**

Different section types fro different applications; the correct type must be chosen, and is then used to create wing's geometry
Modern airlines have aerofoil changing from the root to the tip.

\[ Re = \frac{\rho U c}{\mu} \]
Aerofoil with camber
Types of camber

Zero camber (symmetric)

Positive camber (lift up)

Negative camber (lift down)

Thickness distribution

\[ x_u = x - y_t \sin \theta \]
\[ y_u = y_c + y_t \cos \theta \]
\[ x_l = x + y_t \sin \theta \]
\[ y_l = y_c - y_t \sin \theta \]
General aerofoil specifications:

Low speed
Hobby aircraft, UAV
Less than $500000 \text{Re}$
Selif, Lissaman famous

Subsonic:
Large leading edge nose radius
Performance invariant to small AoA change
No major drag sacrifice
Max mach 0.75/0.8

Transonic
Maximisation of drag divergence mach number $\frac{dC_d}{dM} = 0.1$

Supersonic
$1 < M < 5$ mach number
Reduce wave drag
Small thickness (3.36% for F104)

Natural laminar aerofoil: (First used in P51)
- Lower skin drag
- Minimum pressure point as far downstream as possible
- Manage pressure gradient
- External disturbances (Tollmien-Schlichting waves (instability))

Multi element
- High lift $\rightarrow$ increased weight $\rightarrow$ increased cost
- Single, double and triple element (each increasing in weight and cost)
- But about 0.1 increase in $c_l$ is roughly $1^\circ$ less in angle in approach, so we don’t need as big a landing gear

Morphing
- Change shape
- Alternative to multi element

NACA Aerofoil

NACA 4 series:
- Ground breaking as it parametrised the aerofoil

The geometry equation is:

$$y_t = a_0\sqrt{x} - a_1 x - a_2 x^2 + a_3 x^3 - a_4 x^4$$

- The square root helps with the leading edge nose radius:
\[ R_{LE} = \left[ 1 + \left( \frac{dt}{dx} \right)^2 \right]^{\frac{3}{2}} \approx 1.1019 t_{\text{max}}^2 \]

NACA2412; max camber = 2%; location from leading edge = \( \frac{4}{10} \); maximum thickness \( t = 12\% \)
- Issues with camber line, extreme values of camber, extreme forward camber locations

**NACA 5 series:**
NACA 23012 last 2 digits are thickness as a percentage of the chord. The others are parametrised as coefficients

\[ y_c = C \left( \frac{k_1}{6} (x^3 - 3mx^2 + m^2(3 - m)x) \right) \]

**NACA 6 series:**
NACA 66\(^2\)-215
- 1\(^{st}\) digit indicated 6 series
- 2\(^{nd}\) is the chordwise position of the minimum pressure in 1/10 of the chord for symmetrical aerofoil at \( c_l = 0 \)

**Effects of wing section:**
= angle of zero lift
- Lift curve slope
- Max lift
- Angle of max lift
- Moment around the \( \frac{1}{4} \)

---

Tute 1
Writing reports
- Simplicity and clarity
- Write a few drafts

Lab report Components:
- Abstract
- Introduction
- Methods
- Results
- Discussion
- Conclusion

Structure
Title page
Abstract/summary
Intro
Methodology
Findings/results
Analysis
Summary/conclusion
References
Appendices

- Write shorthand to sound scientific/objective
- Facts and details rather than analysis
- Imply analysis and reasoning without making argument explicit
- Assume reader will read meaning into text
- Good usage, spelling, grammar and punctuation

Components:
Intro: background/objectives, scope and limits, previous work/research; important background information, research aims, (how does lab fit in with body of work)
Method: procedure/material
Results: data, tables, figures, calculations
Discussion: link to intro, interpretation, alternative explanations
Conclusion: summary main point
References: sources
Where to start:
Start with data, not intro

How to display the data to be meaningful and concise

Trends in figures

Connect results analysis to theory

Theory:
- Which research question did you set out to answer
- Expected answer or assumptions
  - Hypothesis
  - Designed to prove

Methods:
- Accurate and complete account of method

Results
- Present data
- State in verbal as well as visual
- Draw attention to key points
- Number/title tables/graphs
- Appendix for raw data or complex calculations

Conclusion:
- Short and to the point
- State what you know
- No new information should be disclosed
- Tie into the introduction

Cover page:
- Neat and clean
- Typed
- Make a good impression
- Include
  - Title of lab
  - Name, SID; lab parternes
  - Date

Technical writing language:
- Impersonal; avoid first person
Common mistakes

- Don’t rewrite lab sheet
  - There are many plotters which do a better job
  - If using matlab, don’t put a grey background
- Don’t use default excel style plots (find better ones)
- Write meaningful figure captions
- Computer spell checks can’t recognise gibberish
- Don’t print table and graph. Graph has information, put table in the appendix
- Negative drag is not a thing‼!
- Incorrect referencing: reference has to appear in the text otherwise it’s a bibliography
- Error analysis (should contain error analysis)
  - Sources of errors not listed (realistic errors)
  - Error bars in results (X and Y)
- Plots
  - Include units, labels
  - Make possible to read
  - Discrete points
  - How to connect the points
- No referencing of figures/tables in text
- Don’t see “as seen in figure below”, say “in figure 3”
- Reference figures/data from external sights (or it’s plagiarism)
- Comparison to literature
  - Not evident
  - Severely lacking
  - Not mentioning who comparing with
- Work on english

Lecture 3. Tuesday, 2 August 2016

\[ L = \frac{1}{2} \rho U^2 C_l = \rho U \Gamma \]

Aerofoils (again)

Leading edge nose radius

- Small leading edge radius has sharp stall break
- Large leading edge radius has gentle stall break

Stall
Increasing angle of attack too much creates separation; so not enough lift created (stall)

Factors that contribute to stall
As angle of attack increases the stagnation point moves further down on the forward part of the aerofoil; making a longer effective upper surface

- This creates friction that increases with travel distance
- Pressure gradient (pressure change)
  - Decrease in pressure from leading edge, which decreases with distance
  - This decreasing pressure tends to induce the flow to move along the surface, promoting the flow direction we want (favourable pressure gradient)

![Figure 1. Drawing illustrating the leading edge geometry.](image)

Beyond this peak in negative pressure we find a reversal

As angle of attack increases the centre of pressure moves forward and unfavourable pressure gradient becomes longer and steeper

- Eventually, the combined effect of the unfavourable pressure gradient and the surface friction becomes greater than the energy available in the airflow
- With not flow over top surface, there is no mechanism to reduce pressure and lift decreases
  - Lift does not go to zero; just a big drag penalty
Reducing stall abruptness:
- Roundess of leading edge acts as a barrier to flow at high angle of attack
- Stall stripes

Leading edge stall
Linear increase in cl until stall • At $\alpha$ just below 15º streamlines are highly curved (large lift) and still attached to upper surface of aerofoil • At $\alpha$ just above 15º massive flow-field separation occurs over top surface of aerofoil → significant loss of lift • Called Leading Edge Stall • Characteristic of relatively thin aerofoils with thickness between about 10 and 16 percent chord
Trailing edge stall

- Progressive and gradual movement of separation from trailing edge toward leading edge as $\alpha$ is increased

Thin aerofoil stall

- Flow separates even at very small $\alpha$
- Initially small regions of flow separate to form separation bubble
- As increased reattachment point moves further downstream until total separation

Bubble increases the camber; which changes the $C_l$ alpha curve
Surface oil visualisation

Lecture 4. Thursday, 4 August 2016

Selecting an aerofoil
- Highest maximum lift coefficient
- Proper ideal or design lift coefficient
- Lowest $c_{\text{d}}$
- Highest L/D ratio
- Highest lift curve slope
- Lowest pitching moment coeff
- Proper stall quality
- Can be structurally reinforced
- Must be able to be manufactures
- Cost
- Other design requirements
- Integration of aerofoils along span

**Maximising lift**
- 2 parameters critical and both dictated by pressure distribution
  - Boundary layer separation
  - Onset of supersonic flow
- Upper surface is most critical
- Try and achieve constant pressure across top surface
- Reduce shock strength and wave drag

**Increasing lift**

\[ L = \frac{1}{2} \rho U^2 S \frac{dC_L}{d\alpha} \]

Take off and landing needs to be augmented

- We have the ability to change \( U \) and \( S \)
High lift devices

1. No flap
2. Plain flap
3. Split flap
4. L. E. slat
5. Single slotted flap
6. Double-slotted flap
7. Double-slotted flap with slat
8. Double-slotted flap with slat and boundary layer suction
9. Not shown - Fowler flap
Common ones are

- Simple flap
  - Increase camber and angle
- Fowler flap
  - Increase camber, angle of incidence and wing area
- Nose flap
  - Increase camber

**Slat**
Reduce velocity over upper side of main wing and increases on the lower side

- Severity of adverse pressure gradient reduced
- Lower pressure on upper side is offset by higher pressure on lower side

**Flaps**
Increase velocity over both surfaces

- Increase speed help again negative pressure gradient
- Effective angle of attack is increased
- Extra circulation created for kutta condition
Effect of Flaps

TYPICAL VARIATION OF FLAP DRAG

$C_L$ vs. $C_D$
Supercritical aerofoils

Slotted

- Reenergise the BL
- Negative camber ahead of slot
- Large amount of positive camber after slot
- Increase in skin friction
- Extremely sensitive to sape

- Keeps flow constant over upper surface to avoid negative pressure gradient
- Reduce shock strength
- Delay drag rise
Standard vs supercritical aerofoil
In supersonic flow, there can also be internal shockwaves inside the flow.
Effect of thickness at trailing edge

Navier Stokes and Potential Flow theory

Equations of motion: Navier stokes equation

- Most complete model of flow in a continuum is NS
- Only a model
- Represents 3 conservations law: mass, momentum and energy
- There are models starting from the Boltzmann equation

Derivation:
Look at infinitesimal fluid element; with velocity $u, v, w$ and length $dx, dy, dz$

Mass (continuity):
- Net mass flow rate is rate of change of mass per time

Momentum:
- Rate of increase in momentum + rate at which momentum leaves = body forces + pressure force + viscous force

Energy:
- Rate of change of energy = flux of heat + rate of work

**Stress tensor:**

Continuity equation

\[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{V}) = 0 \]

**Navier stokes equation**

Tensor notation:

\[ \rho \frac{Du_i}{Dt} = -\frac{\partial \rho}{\partial x_i} + \mu \frac{\partial^2 u_i}{\partial x_i^2} \]

Vector notation:

\[ \rho \left( \frac{\partial \vec{u}}{\partial t} + \frac{1}{2} \nabla \cdot \vec{u} + (\nabla \times \vec{u}) \times \vec{u} \right) = -\nabla p + \mu \nabla^2 \vec{u} \]

Matrix notation:

\[ \rho \left( \frac{\partial \vec{u}}{\partial t} + \nabla \cdot \vec{u} \times \vec{u} \right) = -\nabla p + \mu \nabla^2 \vec{u} \]

**Comments:**
- Most complete form of airflow equation although turbulence not explicitly defined
- Explicit definition of turbulence further complicates the equation by introducing new unknowns, the Reynolds stresses
- The most famous models for turbulence are the \( k \omega; k \epsilon \) and wall model
- No explicit solution
- Equations are: unsteady, non linear, viscous, compressible (nonlinearity is big problem)
**Constant viscosity equations: Incompressible Steady euler equations**

Incompressible: $\nabla \cdot u = 0$; steady: $\frac{\partial}{\partial t} = 0$

$\mu$ constant assumption

$$u \cdot \nabla u = -\frac{1}{\rho} \nabla p$$

$$\nabla \cdot u = 0$$

- Easier to solve than navier stokes; but still requires numerical methods (eg finite difference). Very few analytical solutions (and only for specific cases)

**Flow rotation**

Without viscosity; we can’t apply shear to a little section, so infinitesimal section is irrotational

- Some flows can be idealised as irrotational
- In general, attaches, incompressible, inviscid flow is also irrotational
- The curl of the velocity is zero

**Irrotational flow**

$$\nabla \times \vec{u} = 0$$

**Potential flow**

Irrotationality leads to the simultaneous equations:

$$\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0$$