Topic 3: Wealth, Time & Money (Intro to Financial Math)

Why study Financial Math?

Financial math is about *Valuation*

Value is defined as the *worth of an asset*, the total of what the asset can earn over the period held. The worth of an asset typically comes from net income in the future and terminal value.

- The value established could be a present value or future value

First step is to estimate the components of cash flow streams that arise from financial products and real assets

- E.g. of financial products are loans, shares and bonds
- E.g. of real assets are real estate and equipment (investable, tangible projects)

Ultimately **value is important because it is used to make buying and selling decisions of financial products and real assets**

Value needs to be distinguished from the concept of "Price".

- The price of a financial product or real asset is what it trades at in a market as a function of its supply and demand.
- The value of an asset, i.e. what it is worth, is one factor that influences the supply and demand of the asset and thus it's price. There are other factors that affect supply and demand.

Other factors affecting supply and demand?

- Financial regulation
- Market sentiment, 'emotion' of buyers and sellers at the tine

Value < Price, the asset can be termed "over-priced"

a. A buyer wouldn't buy; they pay more than what the asset can earn.
b. A seller would sell; they receive in price more than the asset can earn if kept.

Value > Price, the asset can be termed "under-priced"

c. A buyer would buy; they pay less than what the asset can earn.
d. A seller wouldn't sell; they receive in price less than what the asset earns if kept.

This process of comparing value to price aids in understanding one of the most widely used and important capital budgeting (investment) metrics: Net Present Value (NPV)

Critical Thinking Conclusion: Why is valuation important in finance?

- *As a buyer: What price should I pay for this investment?* Ans: A price not over the asset's value. The best price is $0, but it is unlikely a seller would sell for that. Value help sets a price ceiling.
- *As a seller*: *What price should I sell this asset for?*Ans: A price over the asset's value. Value help sets a price floor.

The buyer and seller of the same asset can have different present valuations of the same asset- as they can have different discount rates

Time Value of Money

It is said that 'a dollar today is worth more than a dollar tomorrow'; why is that?

Consider if you would prefer to receive $100,000 today or $100,000 next year. Preference is for current consumption, $100,000 today, rather than deferred consumption, $100,000 next year. This is because you can **invest** $100,000 today and earn interest on this amount. Your $100,000 will accumulate interest. The **Future Value** of you $100,000 would include $100,000 plus interest, which is worth more than $100,000. So $100,000 **today** is worth **more** than $100,000 in the future. *For example*

The value of $100,000 investment in one year:
If the interest is 10%, the cost of investment is:
Cost = ($100,000 today) + (10% x $100,000 interest earned) = $110,000 in one year
So, $110,000 is the opportunity cost of spending $100,000 today

The time at which money is earned or paid affects its value or cost

- Money received or earned today, is greater in value to the recipient than if received at a later date
- Money paid today is greater in cost to the payer, than if paid at a later date

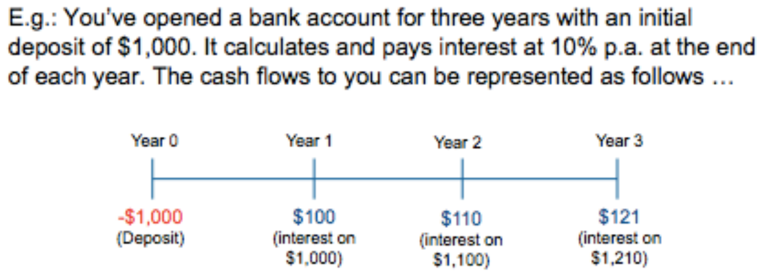
This is intuitively true…

1. Would you prefer to receive $1,000 now *OR*
2. Wait 10 yrs. to receive the $1,000.

We prefer the earlier option because of the **utility** of money

- Utility = satisfaction
- The sooner we get the money the greater our satisfaction – as we can choose to spend/invest that money earlier and vice versa

Cash Flow Timelines



Given that money has different value dependent on the time it is earned or paid, three central rules arise:

- Money can be only be combined and compared if earned at the same time period. (As they have the same time value)
- Calculating a Future Value (FV) given periodic growth on an investment is called **compounding**. Often used to work out what capital accumulates to in the future given re-investment of a periodic return. E.g. …
- An interest bearing deposit account with a bank
- A sinking fund or investment fund, which earns a periodic return and re-invests that return.
- Calculating a Present Value (PV) by diminishing money earned in the future to reflect the time value of money is called **discounting**. We review the reasons for discounting in Slide 16. E.g.…
- In capital budgeting, we discount the cash flows of assets to establish PV today so as to compare value with today's price.
- We can discount cash flows of comparable liabilities to work out the Present Value of cost and identify which is cheaper.

Using timelines is important to understanding the time value of money. Note: We can only add amounts at the **same point in time**!

For example
What is the Future Value of $100,000 invested today over a three-year period at 10% per annum?

Now is time 0, and we are required to find out the amount this $100,000 will accumulate to after investing it for three years at 10% per annum (and not taking out our interest earnings each year).

Drawing a timeline provides a visual aid in solving problems such as these, and is especially useful for more complicated cash flow problems.

Future Value
The general formula for the future value of an investment over *n* periods can be written as:

$$Future\ Value = Present\ Value\ x\ (1+i)^n$$

Where:

- FV is the accumulated future value at the end of period n
- PV is the cash flow at time 0
- I is the interest rate

For example
What is the Future Value of $100,000 invested today at 10% per annum over three years?

The Future Value is $133,100 at the end of the three years. This is greater than $100,000 received at the end of three years.

<u>**Financial Math**</u>
When money is compounded by periodic growth rate, a **future value (FV)** is obtained
A single cash flow, compounded over time is given by:

$$\boldsymbol{FV} = PV\ (1+i)^n$$

Future Value and Compounding
Individuals prefer to consume now rather than later – to encourage individuals to defer consumption, some form of compensation would have to be offered – interest. Compensation in the firm of interest is their Required Rate of Return – this can be compounded, showing how money can grow

When money is discounted, a **present value (PV)** is obtained
A single cash flow discounting over time can be calculated by:

$$\boldsymbol{PV} = \frac{FV}{(1+i)^n}$$

Where i is the interest rate for FV, and the discount rate for PV and n is the number of payments

Effective Annual Interest Rate (EAR)
The effective return that includes the compounding effect of the frequency of payment per annum

$$\boldsymbol{EAR} = (1+\frac{i}{m})^m - 1$$

$$FV = PV \times (1 + \frac{i}{m})^{m \times n}$$

Where I = nominal rate or annual % rate
M = frequency of payment per annum
N = number of years

The greater the frequency of payment (m) the higher the effective return

<u>Example</u>

You have $10,000 to invest for one year and the banks in your area offer the following choices:

1. 6% compounded annually
2. 5.9% compounded daily

Which alternative would you choose? It is easier to decide if you calculate the EARs.

$$\text{a) } EAR = \left(1 + \frac{i}{m}\right)^m - 1 = \left(1 + \frac{0.06}{1}\right)^1 - 1 = 6.00\%$$

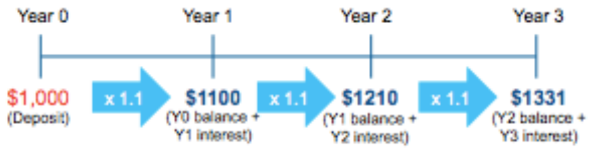
$$\text{b) } EAR = \left(1 + \frac{i}{m}\right)^m - 1 = \left(1 + \frac{0.0590}{365}\right)^{365} - 1 = 6.08\%$$

<u>Cash Flow Patterns</u>

1. **Single/Lump sum cash flow for PV & FV –** one cash flow at time 0 (CF0)
2. **Mixed stream/multiple cash flow for PV & FV –** CF0, CF1, CF2…
3. **Annuities** – multiple cash flows are all equal (CF0=CF1=CF2=CF3)
4. **Perpetuities** - if the annuity goes on forever

Example: Time value of Money – Single Cash Flow FV

You've opened a bank account with an initial deposit of $1,000. It calculates and pays interest at 10% p.a. at the end of each year, for 3 years. What is the future value of your investment?



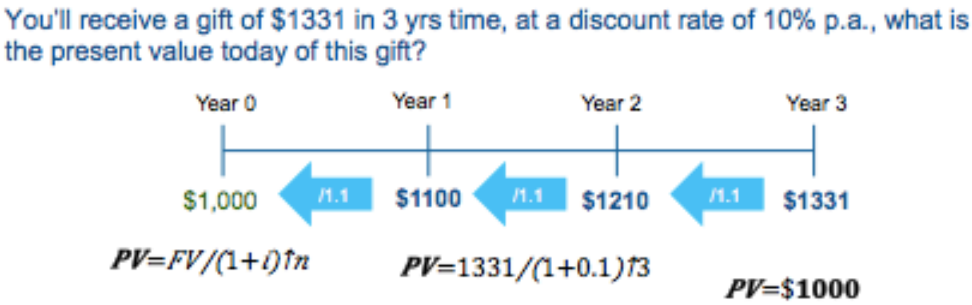
$$FV = PV(1+i)\uparrow n$$

$$FV = \$1000(1+0.1)\uparrow 3$$

$$FV = \$1331$$

<u>ON CALC:</u>	**<u>ON EXAM:</u>**
10 I/YR 3 N 1000 PV → FV	▪ Write out the appropriate formula ▪ Substitute correct values ▪ Work out correct answer (manually or with calculator)

Example: Time value of Money – Single Cash Flow PV

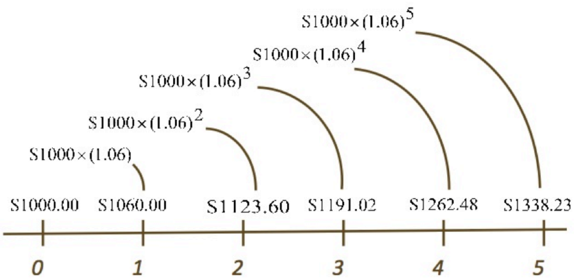


<u>ON CALC:</u>
10 I/YR
3 N
1331 FV
➔ PV

For example
Suppose that you invest $1000 in an account today. Assume that you are investing at a guaranteed fixed rate of 6% per annum over a period of 5 years, how much will this amount grow to by the time the investment account matures? In other words, how much would you have accumulated at the end of the five year period?

$$Future\ Value = Present\ Value\ x\ (1+i)^n$$
$$Future\ Value = 1000\ x\ (1+0.06)^5$$
$$Future\ Value = \$1338.23$$

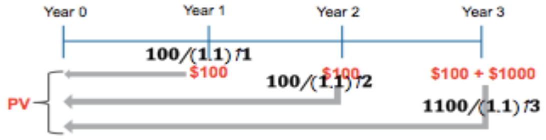
Year	Opening Balance	Interest	Closing Balance
1	$1,000.00	$60.00	$1,060.00
2	$1,060.00	$63.60	$1,123.60
3	$1,123.60	$67.42	$1,191.02
4	$1,191.02	$71.46	$1,262.48
5	$1,262.48	$75.75	$1,338.23



1. EARs apply primarily to FV not PV (uses discount rates which are not compounded)
2. FV is used primarily for assets; PV is common for both assets & liabilities. In FV, you are compounding future earnings by a growth rate (*i*). Still, in liabilities, FV can be used to compound future costs. E.g. the interest you receive on a term deposit has to be paid by the bank. The compounding interest to you is a compounding bank cost

Example: Time value of Money – Multiple Cash Flow PV

What is the present value cost of a $1000 loan charging 10% p.a. interest over 3 yrs, discounted at 10%? For multiple cash flows, create a timeline and set out the cash flows clearly before applying the PV formula to each cash flow individually.



$PV = 100/(1+0.1)^1 + 100/(1+0.1)^2 + 1100/(1+0.1)^3$

$PV = 90.91 + 82.64 + 826.45 = \$1000$

ON CALC:
10 I/YR
3 N
0 CF_J
100 CF_J
100 CF_J
1100 CF_J
NPV (*down PRC*) ➔ *PV*

It is important to distinguish between various types of return:

2. Nominal return: *Also known as the annual percentage return (APR).*

- Effective return (e.g. EAR): *The return that includes the effect of compounding.*
- Real return: *The return that accounts for the erosion of purchasing power due to inflation.*
- Required rate of return: *The minimum return needed. WACC is an example of a RRoR.*
- Expected rate of return: *The anticipated/forecasted return that will be earned on an asset.*

Yield, Return and Discount Rates

The yield or return of an asset is a widely used metric for relative performance

- It expresses the money earned by the asset, as a percentage of the price paid for the asset
- The money earned can be income, capital gains or a combination of both
- The percentage is usually expressed for a time period (p.a.)

Distinguishing between Yield and Discount Rate

Yield is commonly a relative performance measure, in simple form Yield = Earnings/Price

The discount rate is used to calculate present value – it is often the required rate of return and that used to discount future earnings/cash flow to a present value

This discounting of future earnings to a lower present value reflects and incorporates:

- Time value of money
- Inflation
- Risk

Discounting

When converting cash across time, it is important to remember that money in the future is worth less than today, so its price reflects a discount.

- Discount Rate is the appropriate rate to discount a cash flow to determine its value at an earlier time
- Discount Factor is the value today of a dollar received in the future, expressed as $\frac{1}{(1+r)}$

For example

What is the value of $105,000 benefit received one year from now if the interest is 10%?

1. FV is $105,000
2. N is 1
3. I is 10%
4. If $FV = PV(1+i)^n$, then we have $PV = FV/(1+i)^n$
5. PV = $105,000/1.10 = $95,454.55
    a. The value that needs to be invested today at 10% per annum to receive $105,000 in one year

The required rate of return is commonly used as a discount rate in PV calculations.

- It incorporates risk, inflation and the opportunity cost of capital (which considers R_f)
- When using the RRoR to calculate value, future earnings are *discounted* by what they have to earn; which is in effect, paying for RRoR.
- The remaining value is then compared to price to help determine buying or selling decisions.
- In capital budgeting, an RRoR is WACC that serves a common discount rate.

Topic 4: Wealth, Time & Money (Annuity Financial Math)

Annuities

In finance we commonly encounter situations, which call for payments of **equal** amount of cash at **regular intervals** of time over **several time periods**

- One could take out a loan to be repaid in equal regular payments over a finite number of periods
- One could save regular equal amounts over a finite number of periods which accumulate to a lump sum to be take at the end of the last period

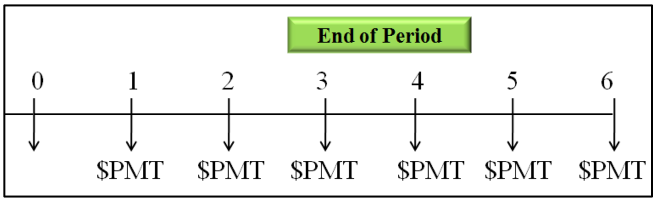
Any financial contract that calls for **equally spaced and level cash flows** (consecutive regular and equal payments) over a finite number of periods is called an **annuity**

- Business, personal, car and home loans, insurance policies, etc.

When valuing annuities, rather than discount/compound each cash flow individually, as each cash flow is the same over time a single formula can be applied

Ordinary Annuities

Most annuities are structured so that cash payments are **paid/received at the end** of the period



A **fixed** amount of money is paid/received at **fixed** intervals of time for a **fixed** period of time

- Also known as *Annuity in Arrears* or *Deferred Annuity*

If nothing is said about the cash flows always assume it is an ordinary annuity → **1st cash flow occurs 1 period after the start of the annuity (END of each period)

Ordinary Annuity Valuation

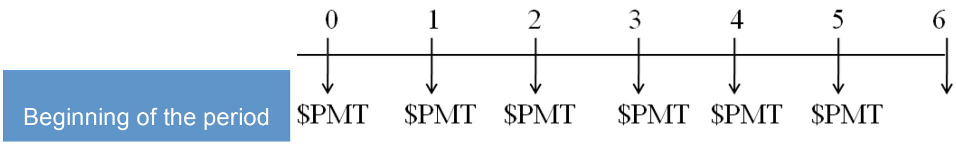
$$PV = \frac{PMT}{i}\left[1 - \frac{1}{(1+i)^n}\right] \quad FV = \frac{PMT}{i}\left[(1+i)^n - 1\right]$$

FV = future value of annuity *PV = present value of annuity* *PMT = cash flow received/paid under the annuity* *N = number of payments* *I = the per-period discount rate (PV) or compound rate (FV) – I and R are used interchangeably*

IMPORTANT: PV of Ordinary Annuities is for the time period before the 1st payment (PMT_1 → PV_0)

Annuity Due (Annuity in Advance)

Annuity where payments are **paid/received** at the **start of each period**



*****For an Annuity Due the 1st Cash flow occurs at the start of the annuity**