36.6: Time Dilation

Imagine a stationary observer with one light clock, where t is the time between reflections on the stationary clocks. In the stationary frame, a moving clock runs slowly and Δt' <Δ t.

$$\boldsymbol{\Delta t'} = \frac{\boldsymbol{\Delta t}}{\sqrt{\boldsymbol{1} - \frac{\boldsymbol{v^2}}{\boldsymbol{c^2}}}}$$

This can be changed to: $\boldsymbol{\Delta t'} = \boldsymbol{\gamma \Delta t}$

Δt': Stationary Observer
Δt: Moving Observer

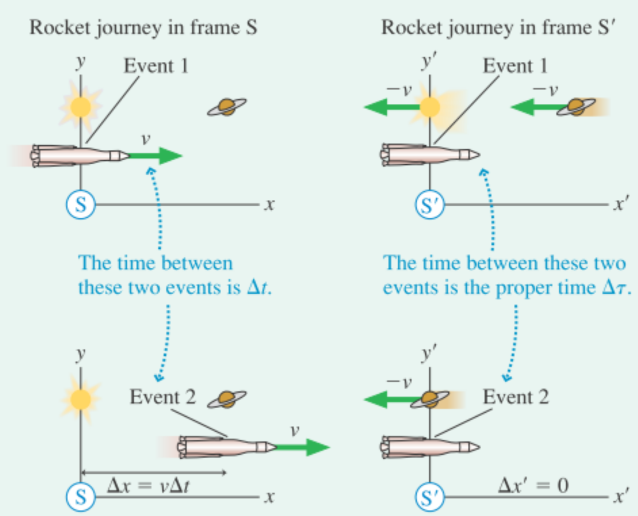
Δt: Proper Time: The time interval measured by a stationary observer in an inertial frame of reference (Event occur at the same point in space)

EXAMPLE 36.5 From the sun to Saturn

Saturn is 1.43×10^{12} m from the sun. A rocket travels along a line from the sun to Saturn at a constant speed of $0.9c$ relative to the solar system. How long does the journey take as measured by an experimenter on earth? As measured by an astronaut on the rocket?

MODEL Let the solar system be in reference frame S and the rocket be in reference frame S′ that travels with velocity $v = 0.9c$ relative to S. Relativity problems must be stated in terms of *events*. Let event 1 be "the rocket and the sun coincide" (the experimenter on earth says that the rocket passes the sun; the astronaut on the rocket says that the sun passes the rocket) and event 2 be "the rocket and Saturn coincide."

FIGURE 36.22 Pictorial representation of the trip as seen in frames S and S′.



VISUALIZE FIGURE 36.22 shows the two events as seen from the two reference frames. Notice that the two events occur at the *same position* in S′, the position of the rocket, and consequently can be measured by *one* clock.

SOLVE The time interval measured in the solar system reference frame, which includes the earth, is simply

$$\Delta t = \frac{\Delta x}{v} = \frac{1.43 \times 10^{12}\ \text{m}}{0.9 \times (3.00 \times 10^8\ \text{m/s})} = 5300\ \text{s}$$

Relativity hasn't abandoned the basic definition $v = \Delta x/\Delta t$, although we do have to be sure that Δx and Δt are measured in just one reference frame and refer to the same two events.

How are things in the rocket's reference frame? The two events occur at the *same position* in S′ and can be measured by *one* clock, the clock at the origin. Thus the time measured by the astronauts is the *proper time* $\Delta\tau$ between the two events. We can use Equation 36.9 with $\beta = 0.9$ to find

$$\Delta\tau = \sqrt{1 - \beta^2}\,\Delta t = \sqrt{1 - 0.9^2}\,(5300\ \text{s}) = 2310\ \text{s}$$

ASSESS The time interval measured between these two events by the astronauts is less than half the time interval measured by experimenters on earth. The difference has nothing to do with when earthbound astronomers *see* the rocket pass the sun and Saturn. Δt is the time interval from when the rocket actually passes the sun, as measured by a clock at the sun, until it actually passes Saturn, as measured by a synchronized clock at Saturn. The interval between *seeing* the events from earth, which would have to allow for light travel times, would be something other than 5300 s. Δt and $\Delta\tau$ are different because *time is different* in two reference frames moving relative to each other.

Twin Paradox
In this imaginary situation, two identical twins compare their view of time. One twin remains on Earth while the other twin undergoes a very fast trip out to distant star & back again.

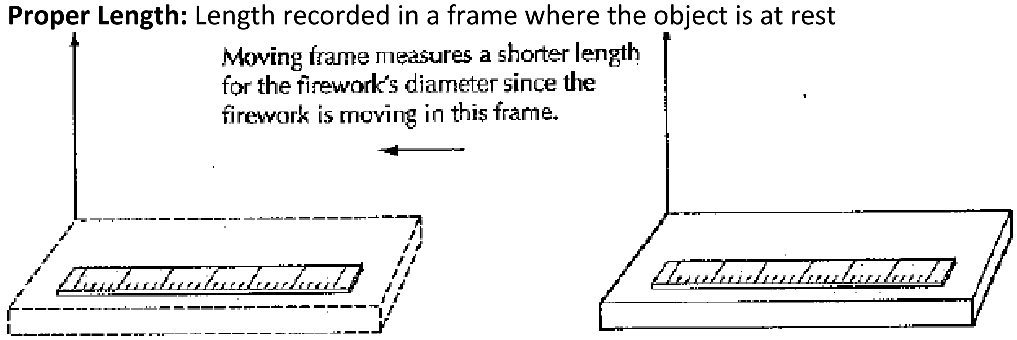
- The twin in space is a moving observer, meaning that the twin on Earth will think that time has been running slowly for the other twin
- When they meet up, the returning twin should have aged less
- Very strange prediction but correct according to time dilation
    - Relativistic effect (time is running differently for them because of relative **velocity** & **NOT** because of the **distance** between them)
    - Difference in ageing is relative (neither twin is getting younger because time is still passing at the normal rate) – It is just that the moving twin thinks that she has been away for a shorter time than recorded by the twin on Earth
- Therefore, the only correct version of time is the twin on Earth, since she has not accelerated

36.7: Length Contraction

Length Contraction
The effect is similar to time dilation. According to a stationary observer, the separation between two points in space2 contracts if there is relative motion in that direction. L_0 is proper length.

- The contraction is in the same direction as the relative motion
- Measured using the formula: $L = \frac{L_0}{\gamma} = L_0\sqrt{1 - \frac{v^2}{c^2}}$

Proper Length: Length recorded in a frame where the object is at rest



EXAMPLE 36.6 The distance from the sun to Saturn

In Example 36.5 a rocket traveled along a line from the sun to Saturn at a constant speed of $0.9c$ relative to the solar system. The Saturn-to-sun distance was given as 1.43×10^{12} m. What is the distance between the sun and Saturn in the rocket's reference frame?

MODEL Saturn and the sun are, at least approximately, at rest in the solar system reference frame S. Thus the given distance is the proper length ℓ.

SOLVE We can use Equation 36.15 to find the distance in the rocket's frame S′:

$$L' = \sqrt{1-\beta^2}\,\ell = \sqrt{1-0.9^2}\,(1.43 \times 10^{12}\ \text{m})$$
$$= 0.62 \times 10^{12}\ \text{m}$$

ASSESS The sun-to-Saturn distance measured by the astronauts is less than half the distance measured by experimenters on earth. L' and ℓ are different because *space is different* in two reference frames moving relative to each other.

Space-time Interval

$$s^2 = c^2(\Delta t)^2 - (\Delta x)^2$$

Spacetime interval s has the same value in all inertial reference frames

EXAMPLE 36.7 Using the spacetime interval

A firecracker explodes at the origin of an inertial reference frame. Then, 2.0 μs later, a second firecracker explodes 300 m away. Astronauts in a passing rocket measure the distance between the explosions to be 200 m. According to the astronauts, how much time elapses between the two explosions?

MODEL The spacetime coordinates of two events are measured in two different inertial reference frames. Call the reference frame of the ground S and the reference frame of the rocket S′. The spacetime interval between these two events is the same in both reference frames.

SOLVE The spacetime interval (or, rather, its square) in frame S is

$$s^2 = c^2(\Delta t)^2 - (\Delta x)^2 = (600 \text{ m})^2 - (300 \text{ m})^2 = 270{,}000 \text{ m}^2$$

where we used $c = 300$ m/μs to determine that $c\,\Delta t = 600$ m. The spacetime interval has the same value in frame S′. Thus

$$\begin{aligned} s^2 = 270{,}000 \text{ m}^2 &= c^2(\Delta t')^2 - (\Delta x')^2 \\ &= c^2(\Delta t')^2 - (200 \text{ m})^2 \end{aligned}$$

This is easily solved to give $\Delta t' = 1.85$ μs.

ASSESS The two events are closer together in both space and time in the rocket's reference frame than in the reference frame of the ground.