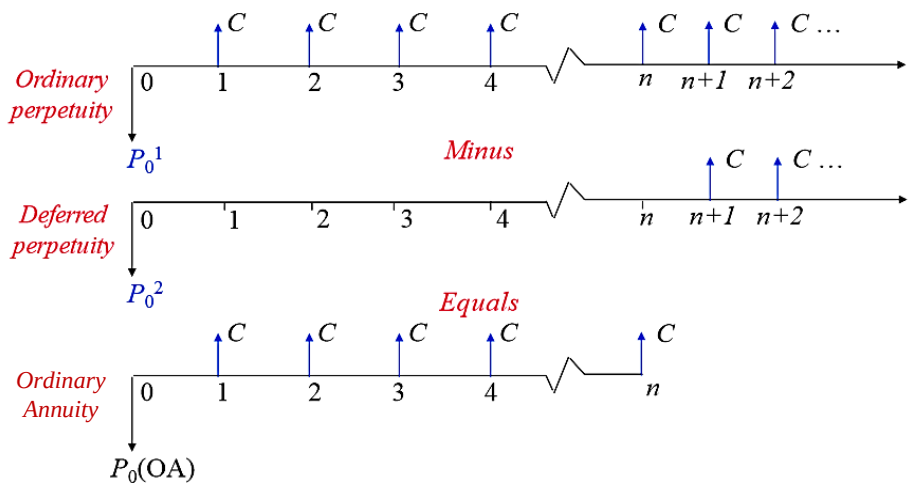
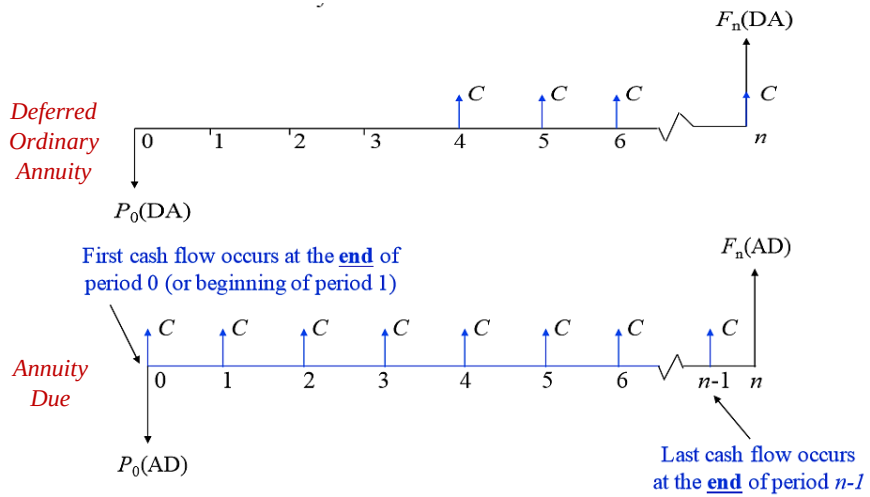
FNCE20001 Business Finance

LECTUER 1-2 Financial Mathematics

Financial instruments	***Equal & periodic cash flow***		***Present value***	***Future value***
	Starts	***Ends***		
Perpetuity	End of $yr\ 1$	∞	$P_0 = C/r$	-
Deferred Perpetuity	End of $yr\ n+1$	∞	$P_n = C/r$ $P_0 = P_n/(1+r)^n$ $= [C/r][1/(1+r)^n]$	-
Ordinary Annuity (OA)	End of $yr\ 1$	End of $yr\ n$	$P_0(OA)$ $= P_0{}^1 - P_0{}^2$ $= [C/r][1 - 1/(1+r)^n]$	$F_n(OA)$ $= P_0(OA)(1+r)^n$ $= [C/r][(1+r)^n - 1]$
Deferred Ordinary Annuity (DA)	End of $yr\ n_s + 1$	End of $yr\ n_e$	$P_{n_s}(DA)$ $= [C/r][1 - 1/(1+r)^{n_e - n_s}]$ $P_0(DA)$ $= P_{n_s}(DA)/(1+r)^{n_s}$	$F_{n_e}(DA)$ $= [C/r][(1+r)^{n_e - n_s} - 1]$
Annuity Due (AD)	Begin of $yr\ 1$ (End of $yr\ 0$)	Begin of $yr\ n$ (End of $yr\ n-1$)	$P_0(AD)$ $= P_1(OA)$ $= P_0(OA)[1+r]$ $= [C/r][1 - 1/(1+r)^n][1+r]$	$F_n(AD)$ $= F_{n+1}(OA)$ $= F_n(OA)[1+r]$ $= [C/r][(1+r)^n - 1][1+r]$





The payment and compounding period are coincident.

[simple annuity] (yearly, semi-annually, monthly)

- $P_0(OA) = [C/r][1 - 1/(1+r)^n]$ if compounded yearly
  $P_0(OA) = [C/\frac{r}{m}][1 - 1/(1+\frac{r}{m})^{nm}]$ where $m = 2$ if compounded semi-annually
  $P_0(OA) = [C/\frac{r}{m}][1 - 1/(1+\frac{r}{m})^{nm}]$ where $m = 12$ if compounded monthly
  Periodic payment (fixed): C
  Interest rate per compounding period: *r/m*
  Number of compounding periods: *nm*

The payment and compounding period are not coincident.

Periodic payment = annually (end of year), compounding period = monthly

Rate equivalence approach:

- Convert the r (nominated p.a. but compounded monthly) into an annualized effective interest rate r_e
- $r_e = (1+\frac{r}{m})^m - 1$ where $\frac{r}{m}$ is the per period interest rate, and m=12 if compounded monthly
- $P_0(OA) = [C/r_e][1 - 1/(1+r_e)^n]$

The loan amortization schedule

Year	**Annual payment**	**Interest paid[1]**	**Principal repaid[2]**	**Principal remaining[3]**
0	–	–	–	$20,000.00
1	$6,309.42	$2,000.00	$4,309.42	$15,690.58 (i)
2	$6,309.42	$1,569.06 (ii)	$4,740.36	$10,950.22
3	$6,309.42	$1,095.02	$5,214.40 (iii)	$5,735.82
4	$6,309.42	$573.58	$5,735.84	$0.00
Totals	$25,237.66	$5,237.66	$20,000.00	

#Step 1: calculate the **"annual payment"** based on the loan amount outstanding.

- $P_0(OA) = [C/r][1 - 1/(1+r)^n]$
- Where C is the periodic payment
  n is the remaining period of cash payment
  r is the interest rate
- loan amount outstanding at any point = principle remaining = the present value of ordinary annuity
  so $P_0(OA)$ = 20 000 000, we generate C using equation

#Step 2: calculate the **"interest paid", "principle repaid"** and **"principle remaining"**

1 Interest paid = Previous period's principal remaining x Interest rate
2 Principal repaid = Annual Loan Payment – Interest paid
3 Principal remaining = Previous period's principal remaining – Principal repaid

Effective interest rate

- $r_e = (1 + r/m)^m - 1$ where m is the compounding frequency in a year
  $r_e = e^r - 1$ if continuously compounding, not common in reality
  In some markets, (e.g. the US and Japan) daily compounding is based on a 360-days year

Compounding interval	***Effective interest rate***
Semi-Annual	$(1 + 0.08/2)^2 - 1 = 8.1600\%$
Quarterly	$(1 + 0.08/4)^4 - 1 = 8.2432\%$
Monthly	$(1 + 0.08/12)^{12} - 1 = 8.3000\%$
Daily	$(1 + 0.08/365)^{365} - 1 = 8.3278\%$
Continuous	$e^{0.08} - 1 = 8.3287\%$
Daily (360-days basis)	$(1 + 0.08/360)^{365} - 1 = 8.4482\%$

LECTURE 18 Debt, Dividends and Taxes I

18.1 Overview

- Corporate taxes t_c = 30%
- Personal taxes t_p are progressive
  - Assessable income – includes salary and wages, investment income, realized capital gains, etc
  - Deductions – include those related to work, managing investments, etc

▼ Tax schedule relevant for the 2015-16 tax year

Taxable Income	*Taxes Payable*
$0 – $18,200	$0
$18,201 – $37,000	19¢ for each $1 over $18,200
$37,001 – $80,000	$3,572 + 32.5¢ for each $1 over $37,000
$80,001 – $180,000	$17,547 + 37¢ for each $1 over $80,000
$180,001 and over	$54,547 + 45¢ for each $1 over $180,000

Note: The above rates do not include the Medicare levy of 2% (subject to reductions) or the Temporary Budget Repair Levy that is payable at a rate of 2% for taxable incomes over $180,000.

  - *Average tax rate* (= Taxes Payable / Taxable Income)
  - *Marginal tax rate* (the tax rate applicable to the last dollar, nominated in table)

18.2 Capital Gains Taxes (CGT)

- For individuals, CGT is the tax that is paid on the net capital gains realized in a particular tax year
- Net Capital Gain = total capital gain realized in that year – (capital loss realized in that year + unapplied net capital losses from earlier years)
  - Capital losses can be *carried forward* to future years and deducted against future realized capital gains
  - No time restriction on how long one can carry forward a net capital loss
- Three methods for computing capital gains taxes
  - The indexation method
  - The discount method
  - The "other" method

	Purchased before 1985, called pre-CGT assets, *exempt from CGT*
September 20, 1985	
	Purchased b/w 1985 and 1999, better of *indexation* and *discount method* (CPI frozen at 68.7 if sold after 1999)
September 21, 1999	
	Purchased after 1999 but held more than 12 months, use *discount method*
12 months before sale	
	Purchased within 12 months, use *'other' method*, pay full tax
Sale date	

Summary of CGT method

	Indexation method	*Discount method*	*"Other" method*
Description of method	Allows an individual to increase the cost base by applying an indexation factor based in CPI up to Sep 1999	Allows an individual to discount the capital gain	Simplest method of subtracting the cost base from capital proceeds
When to use the method	Use for an asset owned for 12 months or more if it produces a better result than the discount method. Use only for assets acquired before Sep 21, 1999	Use for an asset owned for 12 months or more if it produces a better result than the indexation method	Use when as asset is purchased and sold within 12 months (that is, when the indexation and discount methods do not apply)
How to calculate capital gains	Indexed cost base = cost base x indexation factor Net capital gain = capital proceeds – indexed cost base – capital loss	Net capital gain = capital proceeds – cost base – capital losses – CGT discount	Net capital gain = capital proceeds – cost base – capital loss

- Consumption Price Index (CPI)

Year	Quarter ending 31 March	30 June	30 Sept.	31 Dec.
1999	67.8	68.1	**68.7**	
1998	67.0	67.4	67.5	67.8
1997	67.1	66.9	66.6	66.8
1996	66.2	66.7	66.9	67.0
1995	63.8	64.7	65.5	66.0
1994	61.5	61.9	62.3	62.8
1993	60.6	60.8	61.1	61.2
1992	59.9	59.7	59.8	60.1
1991	58.9	59.0	59.3	59.9
1990	56.2	57.1	57.5	59.0
1989	51.7	53.0	54.2	55.2
1988	48.4	49.3	50.2	51.2
1987	45.3	46.0	46.8	47.6
1986	41.4	42.1	43.2	44.4
1985			39.7	40.5

CPI for quarter ending Sep 30, 1999

1. **Indexation method**
   Indexed cost base = cost base x indexation factor
   Net Capital gain = Capital proceeds – Indexed cost base – Capital loss

   For CGT events *before* Sep 21, 1999:

$$\text{indexation factor} = \frac{\text{CPI for quarter when CGT event occurred}}{\text{CPI for quarter in which expenditure was incurred}}$$

   For CGT events *on or after* Sep 21, 1999:

$$\text{indexation factor} = \frac{\text{CPI for quarter ending Sep 30, 1999}}{\text{CPI for quarter in which expenditure was incurred}}$$

   - Note: CGT event means 'sales', Expenditure incurrence means 'purchase'
   - Indexation factors are rounded to *three* decimal places
   - The *higher* this factor (that is, the higher the inflation adjustment) the *higher* the cost base and the *lower* the net capital gain

2. **Discount method**
   Net Capital gain = Capital proceeds – Cost base – Capital loss – CGT discount
   CGT discount = 50% for individuals and trusts
   = $33\frac{1}{3}$% for complying superannuation entities and eligible life insurance companies

3. **The "other" method**
   Net Capital gain = Capital proceeds – Cost base – Capital loss

   Case 1: Purchased in Nov 20, 2014; Sold in Apr 10, 2015 (within 12 months)

Cost of property	$200,000
Stamp duty on purchase of property	$5,000
Solicitor's fees for purchase of property	$2,000
Solicitor's fees for sale of property	$2,000
Agent's commission on sale of property	$4,000
Total cost base	$213,000
Capital proceeds from sale	$250,000
Less Cost base	$213,000
Capital gain	$37,000

* **Indexation method** vs **Discount method** – Choose the method within *lower* capital gains

Case 2: Purchased for $18,000 in Feb 1995; Sold for $25,000 in June 2015

	Indexation method	*Discount method*
Capital proceeds from sale	$25,000	$25,000
Less Cost base of shares	Acquisition cost x indexation factor = $18,000 x 68.7/63.8 = $19,386*	$18,000
Capital gain	$5,614	$7,000
Less CGT discount	-	$3,500
Net capital gain	$5,614	$3,500 (lower)

Case 3: Purchased in Feb 1990

	Indexation method	*Discount method*
Capital proceeds from sale	$25,000	$25,000
Less Cost base of shares	Acquisition cost x indexation factor = $18,000 x 68.7/56.2 = $21,996*	$18,000
Capital gain	$3,004	$7,000
Less CGT discount	-	$3,500
Net capital gain	$3,004 (lower)	$3,500

18.3 Classical Tax System for Dividends

1) Corporate profits are taxed at a flat corporate tax rate t_c, leaving $(1 - t_c)$ to be distributed as dividends
2) Then, dividends received by shareholders are then taxed at the shareholder's personal *marginal tax rate*, t_p

- *Implication?* From one dollar of corporate profit, the shareholder ends up with $(1 - t_c)(1 - t_p)$ dollars of after-personal-tax dividend. Earnings (profits) paid as a dividend are *effectively taxed twice*
- In Australia, a classical tax system operated until *July 1, 1987* be replaced by an imputation tax system

Case 4:

Net operating income	$2,000,000
Interest expense	$400,000
Taxable income	$1,600,000
Tax payable @ 30% t_c	$480,000
Net income	$1,120,000
Dividend received (Net income x 1%)	$11,200
Tax payable (@ 45% t_p assumed)	$5,040
Dividend income after tax	$6,160

An Australian resident who owns 1% of the shares of ABL will receive a dividend of $11,200

- Share of the total taxes paid by firm = 1% x $480,000 = $4,800 (per 1% shares)
- Personal tax payable = $5,040 (per 1% shares)
- Share of the taxable income = 1% x $1,600,000 = $16,000 (per 1% shares)
- The shareholder's effective tax rate = tax payable / taxable income = ($4,800 + $5,040) / $16,000 = 61.5%
- After-personal-tax dividend on a *dollar* of corporate profit = $(1 - t_c)(1 - t_p)$ = $0.385
  Effective tax rate = $1 - (1 - t_c)(1 - t_p)$ = 61.5%

18.4 Imputation Tax System for Dividends

- Taxes paid at the firm and individual levels are treated in an *integrated* manner
- Tax paid by the firm on its earnings are *imputed* (or attributed) to s/h (aka only taxed once in personal level)
- Tax paid by the firm is allocated to shareholders via *"franking credits"* attached to the dividends paid
  - *Franking credits* are treated as both (imputed) personal income *and* (imputed) taxes paid on that income. Franking credits can be used by s/h to fully (or partly) offset tax payable on the dividends
- Eliminate the double taxation of profit, *effectively taxed at the marginal personal tax rate*

Case 5: marginal personal rate is 45% > the corporate tax rate 30%

Net operating income		$2,000,000
Interest expense		$400,000
Taxable income		$1,600,000
Tax payable @ 30% t_c		$480,000
Net income		$1,120,000
Dividend received Div = Net income x 1%	$11,200	Row 1
Franking credit = $Div[t_c/(1 - t_c)]$	$4,800	Row 2
Grossed-up dividend = $Div/(1 - t_c)$	$16,000	Row 3 = 1+2
Tax liabilty (@ 45% t_p)	$7,200	0.45 x Row 3
Less Franking credit	$4,800	Row 5 = Row 2
Tax payable on dividend	$2,400	Row 4 – 5

- Share of the total taxes paid by firm = 1% x $480,000 = $4,800 = Row 2 (same with case 4)
- Personal tax payable = $2,400
- Share of the taxable income = 1% x $1,600,000 = $16,000 = Row 3 (same with case 4)
- The shareholder's effective tax rate = ($4,800 + $2,400) / $16,000 = 45% = personal tax rate

Case 6: marginal tax rate is 30% = the corporate tax rate 30%

Dividend received Div = Net income x 1%	$11,200	Row 1

Franking credit = $Div[t_c/(1-t_c)]$	$4,800	Row 2
Grossed-up dividend = $Div/(1-t_c)$	$16,000	Row 1+2
Tax liabilty (@ 30% t_p)	$4,800	0.45 x Row 3
Less Franking credit	$4,800	Row 5 = Row 2
Tax payable on dividend	$0	Row 4 – 5

- Personal tax payable = 0
- Effective tax rate = ($4,800 + $0) / $16,000 = 30% = personal tax rate

Case 7: marginal tax rate is 19% < the corporate tax rate 30%

Dividend received Div = Net income x 1%	$11,200	Row 1
Franking credit = $Div[t_c/(1-t_c)]$	$4,800	Row 2
Grossed-up dividend = $Div/(1-t_c)$	$16,000	Row 1+2
Tax liabilty (@ 19% t_p)	$3,040	0.45 x Row 3
Less Franking credit	$4,800	Row 5 = Row 2
Tax payable on dividend	-$1,760	Row 4 – 5

- Personal tax payable = -$1,760
- Effective tax rate = ($4,800 – $1,760) / $16,000 = 19% = personal tax rate
- a fully franked dividend will result in excess tax credits which can be used to reduce the tax payable on other income, or refunded if it cannot be used

Implications?

- The imputation tax system eliminates the *double taxation* of corporate earnings (profits)
- Each s/h is effectively taxed at their marginal personal tax rate
- *Caveat:* the analysis is based on the assumption that the firm pays *all* of its after tax *earnings as dividends* and that the dividends are *fully franked*