

**This page may be detached**

CONSTANTS AND UNITS

<i>Quantity</i>	<i>Symbol</i>	<i>Value</i>	<i>Units</i>
1 arc second	1''	$4.85 \times 10^{-6}$	radians
1 astronomical unit	AU	$1.5 \times 10^{11}$	m
1 light year	ly	$9.45 \times 10^{15}$	m
1 parsec	pc	3.26	ly
1 parsec	pc	206265	AU
1 parsec	pc	$3.09 \times 10^{16}$	m
speed of light	$c$	$3 \times 10^8$	$\text{m s}^{-1}$
solar luminosity	$L_{\text{Sun}}$	$3.8 \times 10^{26}$	W
solar mass	$M_{\text{Sun}}$	$1.98 \times 10^{30}$	kg
solar radius	$R_{\text{Sun}}$	$6.96 \times 10^8$	m
Jupiter's mass	$M_{\text{Jupiter}}$	$1.89 \times 10^{27}$	kg
Jupiter's radius	$R_{\text{Jupiter}}$	$7.15 \times 10^7$	m
Earth's mass	$M_{\text{Earth}}$	$5.97 \times 10^{24}$	kg
Earth's radius (equatorial)	$R_{\text{Earth}}$	$6.38 \times 10^6$	m
Moon's mass	$M_{\text{Moon}}$	$7.35 \times 10^{22}$	kg
Moon's radius	$R_{\text{Moon}}$	$1.74 \times 10^6$	m
speed of light	$c$	$3 \times 10^8$	$\text{m s}^{-1}$
Planck's constant	$h$	$6.62 \times 10^{-34}$	J s
Stefan-Boltzmann constant	$\sigma$	$5.67 \times 10^{-8}$	$\text{J m}^{-2} \text{s}^{-1} \text{K}^{-4}$
Boltzmann constant	$k$	$1.38 \times 10^{-23}$	$\text{J K}^{-1}$
Gravitational constant	$G$	$6.67 \times 10^{-11}$	$\text{m}^3 \text{kg}^{-1} \text{s}^{-2}$
Hubble constant	$H_0$	75	$\text{km s}^{-1} \text{Mpc}^{-1}$
Mass of proton	$m_p$	$1.673 \times 10^{-27}$	kg
Mass of ${}^4_2\text{He}$ nucleus	$m_{\text{He}}$	$6.645 \times 10^{-27}$	kg
1 Joule	J	$1 \text{ kg m}^2 \text{ s}^{-2}$	
1 Watt	W	1 J/s	
1 Newton	N	$1 \text{ kg m s}^{-2}$	

GEOMETRY

Circumference of a circle =  $2 \pi r$

Area of a circle =  $\pi r^2$

Area of a sphere =  $4 \pi r^2$

Volume of a sphere =  $(4/3) \pi r^3$

Eccentricity of an ellipse  $e = \sqrt{1 - \frac{b^2}{a^2}}$

EQUATIONS

$$a = \frac{v_f - v_i}{t}$$

$$v = \frac{s}{t}$$

$$c = f \lambda$$

$$E = h f$$

$$\frac{v}{c} = \frac{\Delta \lambda}{\lambda}$$

$$\lambda_{\max}(\text{m}) = \frac{2.9 \times 10^{-3}}{T}$$

$$E = m c^2$$

$$F = \frac{G M_1 M_2}{d^2}$$

$$F = m g$$

$$F = m a$$

$$F = \frac{L}{4 \pi R^2}$$

$$L = 4 \pi R^2 \sigma T^4$$

$$L = 0.57 \times 10^{-7} A T^4$$

For the main sequence:

$$L \sim M^3$$

$$t_{\text{age}} \sim \frac{M}{L}$$

$$P^2 = \frac{4 \pi^2}{G (M_1 + M_2)} a^3$$

$$P^2 = a^3$$

$$\rho = \frac{M}{V}$$

$$p = m v$$

$$p_{\text{angular}} = m v r$$

$$E_{\text{kinetic}} = \frac{1}{2} m v^2$$

$$E_{\text{potential}} = \frac{G M m}{R}$$

$$V^2 = \left( \frac{G M}{R} \right)$$

$$R = \frac{1.22 \lambda}{D}$$

$$\text{focal ratio} = \frac{f_{\text{objective}}}{d_{\text{objective}}}$$

$$P_{\text{light}} \sim \text{area} \sim d^2$$

$$\text{Mag} = \frac{f_{\text{objective}}}{f_{\text{eyepiece}}}$$

$$FOV_{\text{scope}} = \frac{FOV_{\text{eyepiece}}}{\text{Mag}}$$

$$\text{Quantum efficiency} = \frac{\text{Amount of light detected}}{\text{Amount of light incident}}$$

$$d = \frac{1}{p}$$

$$M_{\text{planet}} = \frac{v_{\text{star}} \times M_{\text{star}}^{2/3} \times P^{1/3}}{(2 \pi G)^{1/3}}$$

$$\text{fraction of dip} = \frac{\pi r_{\text{planet}}^2}{\pi R_{\text{star}}^2}$$

$$R_{\text{Sch}} = \frac{2 G M_{\text{BH}}}{c^2}$$

$$a = \frac{2 G M d}{R^3}$$

$$\lambda(\text{at redshift } z) = \frac{1.1 \text{ mm}}{1 + z}$$

$$1 + z = \frac{\lambda_{\text{observed}}}{\lambda_{\text{rest}}}$$

$$v_r = H_0 \times d$$

$$t_{\text{still}} = \frac{t_{\text{moving}}}{\sqrt{1 - \frac{2.8}{r}}}$$

## Topic 1: Foundations of Astronomy

### Numbers and units

#### 1.1 MEASUREMENT OF DISTANCE AND ANGLES

Powers of 10

The universe is all space, time, matter and energy. We need some way of handling the very big numbers involved.

$$10^{-6} \text{ m} = 1 / 1000000 \text{ m} = 1 \text{ } \mu\text{m}$$

$$10^{-5} \text{ m} = 1 / 100000 \text{ m}$$

$$10^{-4} \text{ m} = 1 / 10000 \text{ m}$$

$$10^{-3} \text{ m} = 1 / 1000 \text{ m} = 1 \text{ mm}$$

$$10^{-2} \text{ m} = 1 / 100 \text{ m} = 1 \text{ cm}$$

$$10^{-1} \text{ m} = 1 / 10 \text{ m} = 10 \text{ cm}$$

$$10^0 \text{ m} = 1 \text{ m}$$

$$10^1 \text{ m} = 10 \text{ m}$$

$$10^2 \text{ m} = 100 \text{ m}$$

$$10^3 \text{ m} = 1000 \text{ m} = 1 \text{ km}$$

$$10^{27} \text{ m} = \text{most distant observed objects in the universe.}$$

#### 1.2 UNITS IN ASTRONOMY

Angles are measured in:

- Degrees of arc,
- Arcminutes (**arcmin** or  $''$ ), and
- Arcseconds (**arcsec** or  $'$ )

Distance is measured in:

- Astronomical units, **AU**, where the distance from the Sun to the Earth is 1 AU . . . particularly useful for distances of objects in our solar system,
- Light years, **ly**, where 1 ly is the distance light will travel in one year in empty space . . . particularly useful for distances between stars, and figuring out how long it would take the Starship Enterprise to get from one to another,
- Parsec, **pc**, the distance at which one AU perpendicular to the observer's line of sight subtends an angle of one second of arc, or  $1/3600^\circ$ , and
- Kiloparsec, **kpc**, 1000 parsecs . . . particularly useful for the most distant objects, other galaxies.

Mass is measured in:

- Earth masses,  $M_{\oplus}$ , and Solar masses,  $M_{\odot}$

Luminosity is measured in:

- Solar luminosity,  $L_{\odot}$ , relating the light output of stars and galaxies to that of our Sun.

Time is measured in:

- Seconds, **s**, Minutes, **min**, Hours, **h** or **hr**,
- **Days** (rotation period of the Earth), and
- Years, **yr** (orbital period of the Earth).

### 2 Movements across the sky

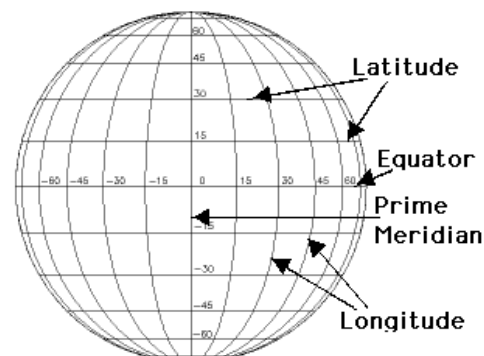
#### 2.1 LONG/LAT AND ALT/AZ

On Earth we use directions **north**, **south**, **east**, and **west** which rely on the rising and setting Sun, and **longitude** and **latitude**.

The **latitude** of the Earth's equator is  $0^\circ$  and we measure north or south of the equator; eg Melbourne's latitude is  $38^\circ$  South.

The **longitude** of Greenwich, England and the great circle through the north and south poles and Greenwich, is  $0^\circ$ . We measure east or west of Greenwich (eg, Melbourne's longitude is  $145^\circ$  East).

The **meridian** is an imaginary circle passing through the north and south poles and an observer's location. The meridian passing through Greenwich is the **prime meridian**.



## APPARENT MOVEMENT OF STARS

Stars that travel in circles around the South celestial pole without dipping below the horizon are called **circumpolar**. The particular stars that are circumpolar depend on the observer's latitude. At the north and south poles all stars are circumpolar! At the equator no stars are circumpolar; they all rise and set.

*What's the highest point that stars reach during the night? What's the highest point that the Sun reaches during the day? If they rise somewhere in the east and set somewhere in the west, then the highest point is called the **meridian transit**.*

**Q1. Which of the following correctly describes the meridian in your sky?**

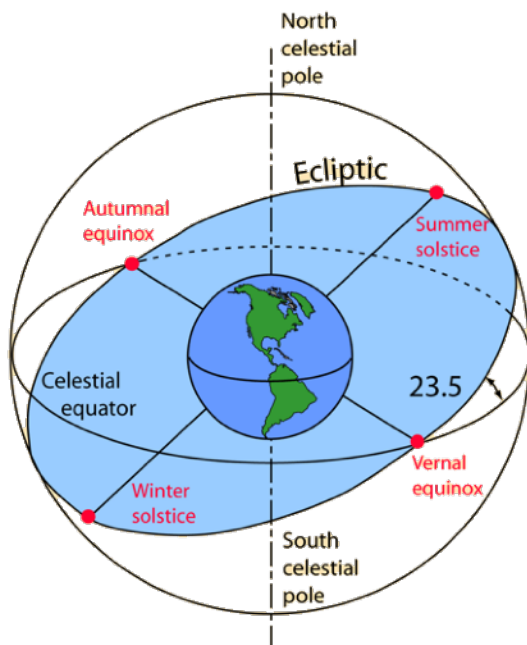
*a half-circle extending from your horizon due north, through your zenith, to your horizon due south*

**Q2. The time between rising and setting of a star**

*depends on the observer's latitude.*

## 2.4 MOVEMENTS OF THE SUN

It **looks** as though the Sun rises and sets (orbits the Earth) every day. It also **looks** as though the Sun moves around the celestial sphere each year, along an imaginary path known as the **ecliptic**.



## SOLSTICES AND EQUINOXES

The Sun moves along the ecliptic, crossing the celestial equator twice each year. On the day of each crossing, all observers on Earth will have 12 hours of daylight and 12 hours of darkness. Those days are called the **equinoxes**: March 21 and September 22.

There must also be a day on which the southern hemisphere has its longest day (and the northern its shortest day) – our (southern) **summer solstice, December 21** – and a day on which the southern hemisphere has its shortest day (and the northern its longest day) – our (southern) **winter solstice, June 21**.

**Q3. Why does the position of the Sun on the celestial sphere change over the course of a year?**

*Because the Earth orbits the Sun over the year, the Sun appears to move on the celestial sphere against the background of the fixed stars.*

**Q4. We see that the Sun is in two different R.A./Dec positions at different times of the year, despite being the same time of day. Why is that?**

*Daylight savings (Sun crosses meridian earlier)*

## Gravity and motion

- *How did ancient Greeks explain planetary motion?*

### 1.1 OLD WAYS: PTOLEMAIC, GEOCENTRIC MODEL

Ptolemy's model had Earth as at the centre of the universe while moon, sun, planets and fixed stars revolved around it.

Facts on the motion of planets:

- the sun, moon and planets all lie in a plane containing Earth
- Planetary motion takes place along a line and are roughly confined to the ecliptic
- Planets moved from West to East
- Planets from time to time will reverse its motion and briefly move from East to West 'retrograde motion'

The Ptolemaic model was composed of circles rotating with constant velocity, with each planet ascended to be attached to a rotating circle (epicycle), whose centre was carried along a second rotating circle (deferent).

Epicycle: meaning circle moving on another circle, was a geometric model used to explain the variations in speed and direction of the apparent motion of the Moon, Sun, and planets.

Deferent: a circle centred on the earth around which the centre of the epicycle was thought to move.

Eccentric: Earth was not placed at the centre of the deferent, but off to one side.

The orbital **eccentricity** of an **astronomical** object is a parameter that determines the amount by which its orbit around another body deviates from a perfect circle. A value of 0 is a circular orbit, values between 0 and 1 form an elliptical orbit, 1 is a parabolic escape orbit, and greater than 1 is a hyperbola.

Equant: The Deferent rotated uniformly, but not with respect to Earth or to its centre, that with yet another off-centre point 'Q'

### 1.2 A NEED FOR A NEW MODEL: THE HELIOCENTRIC SOLAR SYSTEM MODEL

Copernicus demonstrated the mathematical validity of a model with the Sun at the centre of the universe. Astronomers started to take seriously the possibility that geocentrism was wrong.

Copernicus' model assumed circular motion and contained epicycles, equants and deferents.

*Tycho Brahe* was the first scientist to perform high-accuracy measurements. His observations of the motions of the planets provided the basis for Kepler's model of the solar system.

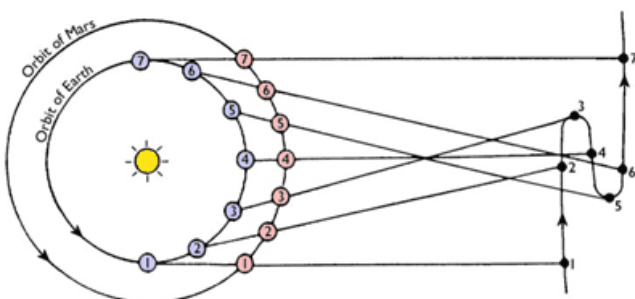
- *What are three Kepler's laws?*

### 1.3 KEPLER'S LAWS

Kepler was employed by Brahe to produce a new model of the Solar System based on his observations. Kepler's model of the solar system rejected the idea of circular motion.

He discovered three laws of planetary motion: planets moved in elliptical orbits, like the planet to the sun sweeps out equal areas in equal times, and the period of an orbit squared is proportional to its semi-major axis cubed.

Kepler's laws apply to planets orbiting the Sun, to moons or other satellites orbiting planets; to stars orbiting each other. Kepler produced **empirical relationships**; others made sense of them and fitted them into theories.



**Retrograde motion** is the motion opposite to the direction of prograde, planetary orbit. Therefore, retrograde motion is in the direction of West to East. However, this reverse motion is only apparent as it is an illusion caused by the moving Earth passing the outer planets in their orbits.

### Kepler's first law, 1609

**The orbit of a planet about the Sun is an ellipse with the Sun at one focus.**

This law applies to every orbiting body.

An ellipse has two foci, the sun, and remarkably nothing occupies the other. Depending on the distance between the two foci, the *eccentricity*, the size of ellipse will change. The larger the eccentricity, the farther apart they are.

When the two foci are on top of each other, we get a circle - an ellipse of zero.

A consequence of this law is that as a planet orbits, it moves closer to and farther from the sun (*not the cause of seasons*).

From his data he found two more relationships by which to describe planetary motion:

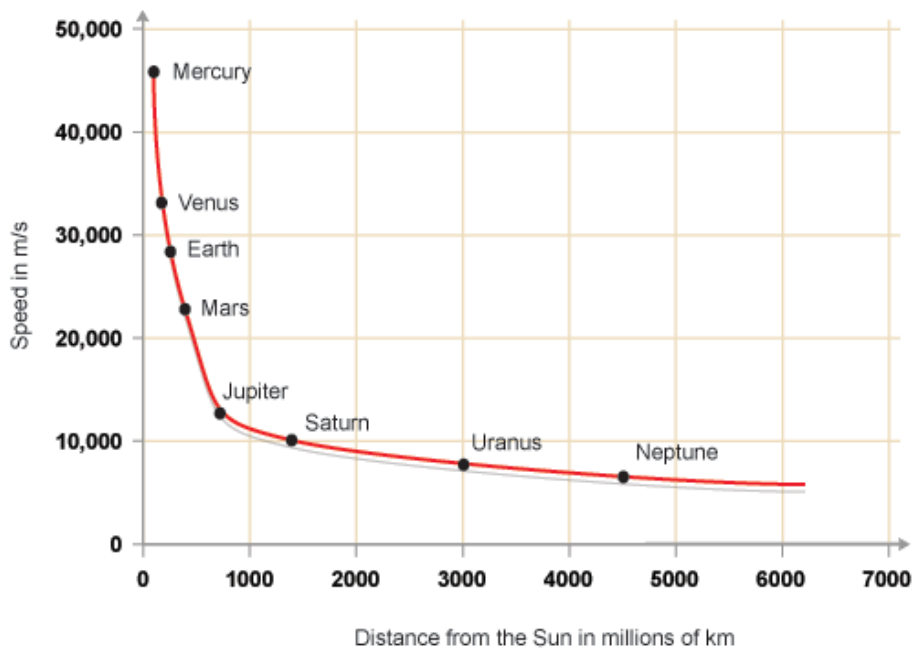
### Kepler's second law

**A line joining a planet and the Sun sweeps out equal areas in equal intervals of time.**

This law applies to every orbiting body.

In order that the various triangles have equal area, their bases must be unequal.

These bases at the distances travelled by the planets in these intervals of time, therefore this law tells us how the body speeds up and slows down as it over the sun—implying that the body moves rapidly when close to the sun, and slowly when far from it.



### Kepler's third law, 1619

**The square of a planet's sidereal period around the Sun is directly proportional to the cube of the length of its orbital semi-major axis**

The third law is concerned with the time required to complete one orbit around the sun - a 'year'. The farther a planet is from the sun, the longer it takes to orbit.

If we call the planet's period '*p*' the length of time required to complete an orbit/year - then this law states that the period squared is **proportional** to the semi-major axis '*a*' cubed.

$$P^2 \propto a^3 \quad \text{or} \quad P^2 / a^3 = K \quad (1)$$

A convenient unit for '*a*' is the astronomical unit (AU) - the average distance between the Earth and the Sun. The AU is the semi-major axis of Earth's orbit.

'*p*' - years

## Measuring the stars

Many cases they can measure the property of the stars in the same way we measure it for the sun.

- Luminosity: if we know the stars distance, measure its apparent brightness/Flux and use the inverse square law.
- Temperature: measure the Spectrum and use the Wein law.
- Composition: measure the spectrum.

The distances to the stars using parallax

- Observer start over the course of the year
- It's apparent motion tells us Its distance
- Parallax formula:  $D = 1/p$  (D= distance in parsecs, p= apparent motion over a year in seconds of arc)

Measuring the sizes of stars

- Even though through a telescope they are too small to be resolved
- They can be determined from the Stefan-Boltzmann law once we have measured a stars luminosity and temperature.

Measure the masses of stars

- Use binary stars
- Observe the orbits of both stars and determine the masses using a generalisation of Newton's mass formula.
- Measure the rotation of the stars by means of the Doppler effect.

## 2.1 Distance and parallax

### Distance to nearby stars: Stellar Parallax

Stars that are relatively close to us (300 ly or less) will appear to move against the more distant 'background' stars **as the Earth orbits the Sun.**

Stellar parallax occurs because we see stars from different vantage points as Earth orbits the Sun. This causes the positions of nearby stars to appear to shift relative to the positions of more distant objects.

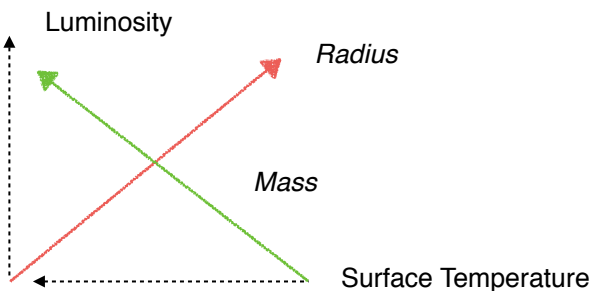
- Parallax is the observed shifting of a star to and fro of a star over the course of the seasons
- It predicts that stars in the ecliptic should appear to move along straight lines and those perpendicular to the ecliptic should appear to move in circles
- We define a star's parallax angle as half the angular separation between the two endpoints of a star's angular motion

Eg. A star with a parallax angle of 1/20 arcsecond is 20 parsecs away

### Hertzsprung-Russell diagram

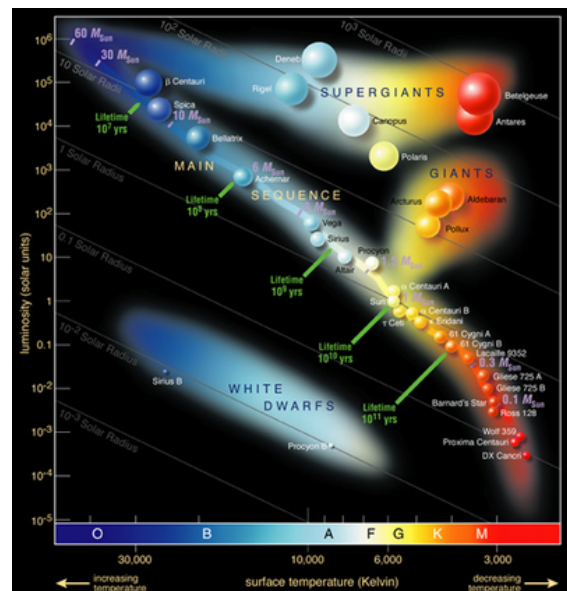
A hotter star emits more light at all wavelengths than a cooler star, so generally we expect that a hotter star should have a higher luminosity than a cooler star.

- An HR diagram plots the luminosity and temperature of stars (do not need to know a star's mass to plot it)
- Main sequence
- Giants and super giants
- White dwarfs
- Spectral class



H-R diagram depicts:

- Temperature
- Luminosity
- Colour
- Radius
- Spectral type



## Stellar lives

### 1.1 Main sequence stars

#### Where do stars form?

- Stars form in dark clouds of dusty gas in interstellar space.
- The gas between the stars is called the interstellar medium.
- We can determine the composition of interstellar gas from its absorption lines in the spectra of stars.
- All stars are born with the same basic composition, yet stars can differ greatly in appearance. Two basic factors that are most important in determining the current appearance of a star are **its mass and its stage of life**.

#### Molecular clouds

Most of the matter in star-forming clouds is in the form of molecules (H<sub>2</sub>, CO, etc)

These molecular clouds have a temperature of 10-30K and a density of about 300 molecules per cubic centimetre.

Most of what we know about molecular clouds comes from observing the emission lines of carbon monoxide (CO).

**Interstellar dust:** *tiny solid particles of interstellar dust block our view of stars on the*

#### Why do stars form?

- Gravity can create stars only if it can overcome the force of thermal pressure in a cloud due to:
  - Compression from a young star's stellar wind
  - Compression from a supernova explosion, powerful stellar winds
  - Compression while passing through a spiral arm

#### Life tracks for different masses

On an H-R diagram, stellar masses can be determined for main sequence stars but not for other types of stars

- **More massive stars live much shorter lives than less massive stars.**
- Models show that sun required about 30 million years to go from protostar to main sequence.
  - Higher-mass stars form faster.
  - Lower-mass stars form more slowly.

*So when does star's life start?* When the hydrogen starts fusing into helium due to **gravity and gas pressure (hydrostatic)**

**Note:** The sun is in a **hydrostatic equilibrium** whereby there is a balance within the Sun between the outward push of pressure and the inward pull of gravity.

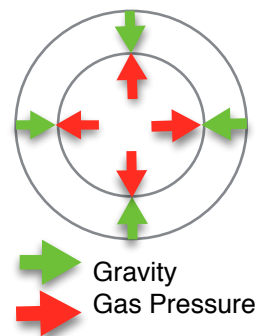
## 2 Low-mass stars: post-main sequence evolution

### 2.1 The giant phase

The main source of energy for a star as it grows in size to become a red giant is hydrogen fusion in a shell surrounding the central core.

#### **After main sequence: running out of hydrogen - red giant stage**

- The star has fused all its hydrogen into helium it has not run out of fuel, as helium is a fuel.
- This new fuel cannot be used until the star has grown hotter.
  - This is because the nuclei that undergo nuclear reactions repel one another.
  - It takes high temperatures to overcome the repulsion between hydrogen nuclei is even higher temperatures between helium nuclei.
- Once the hydrogen fuel is used up, The inner regions of the star contract and heat until the helium commences reacting.
- As this happens the outer regions expand into the star becomes a red giants.
  - Such a star convert helium into carbon.
  - Its surface is cooler than that of the main sequence star.
  - It is larger and more luminous than a main sequence star.
    - When the sun does this the earth will grow too hot to support life leading to the end of the world.





## 1 The Milky Way

We know that our galaxy, the Milky Way, has different components: bulge, disk and halo. Each of these components gives a clue about the formation of our galaxy, as well as about nature of galaxies in general.

### How do we measure the distances to galaxies?

Recall what we learned so far about finding distances:

- for determining distances in the Solar system, we can use orbital periods of planets (or radar measurements)
- for determining distances to nearby stars, we can use parallax (up to around 500 pc)
- for determining distances of the stars further away, we can use spectroscopic parallax (i.e., HR diagram) - up to around 10 kpc

*What does our galaxy look like?*

- Dusty gas clouds obscure our view because they absorb visible light.
- This is the interstellar medium that makes new star systems.
- We see our galaxy edge-on.
- Primary features: disk, bulge, halo, globular clusters
- If we could view the Milky Way from above the disk, we would see its spiral arms

Where do stars tend to form in our galaxy?

Spiral arms are waves of star formation

1. Gas clouds get squeezed as they move into spiral arms
2. Squeezing of clouds triggers star formation.
3. Young stars flow out of spiral arms

### 1.2 Formation of the Milky Way

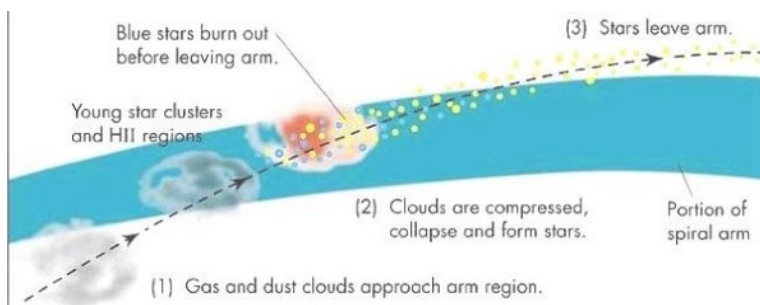
- Our galaxy formed from a cloud of intergalactic gas.
- Halo stars formed first as gravity caused gas to contract.
- Remaining gas settled into a spinning disk.
- Stars continuously form in disk as galaxy grows older.
- Detailed studies show that halo stars formed in clumps that later merged.

### 1.3 Spiral arms

**Self-propagating star formation** causes and maintains flocculent ("fleecy") spiral galaxies:

1. Formation of massive stars in dense interstellar clouds in the disk.
2. Radiation pressure from these hot massive stars compresses the local interstellar gas.
3. These massive stars explode as supernovae, with the shock waves compressing the local interstellar medium, and leading to growth in the star forming region.
4. Differential rotation of the galaxy drags the inner edges (of the young region) ahead of the outer edges, and the new stars spread in the form of a spiral arm highlighted with bright OB stars and nebulae.

**Spiral density waves** cause and maintain grand-design spirals.



Spiral density waves travel through a galaxy, in a manner analogous to ripples in a pond. With water waves the water moves up and down (and back and forth) only locally – no water actually is transported with the wave pattern. The same is true of sound waves, and density waves in general.

### 1.6 Galactic rotation

**How do stars orbit in our galaxy?**

- Stars in the disk all orbit in the same direction with a little up-and-down motion.
- Orbits of stars in the bulge and halo have random orientations.
- The Sun's orbital motion (radius and velocity) tells us **the mass within** Sun's orbit:

$$1.0 \times 10^{11} M_{\text{Sun}} \quad \text{--> that is } 1.9 \times 10^{41} \text{ kg}$$

## Topic 4: Galaxies and Cosmology

### 1 Galaxy types

Type E - Ellipticals

#### **BRIGHTER AND REDDER**

- Show little internal structure
- Elliptical in shape
- No disks, spiral arms, or dust lanes
- Elliptical galaxies are bigger than Spiral galaxies.
- Mostly made of older stars. This is because they are the oldest, having been through the most mergers.
- Brightest stars are red and tend to concentrate in a red sequence.
- Ellipticals are largest, as they have the brightest magnitudes, and they have had the most mergers.

Type S - Spirals

#### **FAINTER AND BLUER**

- Spiral galaxies have three main components: a bulge, disk, and halo (see right).
- The bulge is a spherical structure found in the centre of the galaxy. This feature mostly contains older stars.
- The disk is made up of dust, gas, and younger stars.
- The disk forms arm structures through differential rotation.
- New stars form in the spiral arms.
- The vast majority of spiral galaxies concentrate in a blue cloud between colour 1.25 and 2.0.
- Spirals merge to become ellipticals.

↳ *The Milky Way is thought to have a weak bar, and is classified by some as SBbc (part way between SBb and SBc).*

Type I - Irregulars

- Show an irregular, often chaotic structure.
- Little evidence of systematic rotation.

#### **How are galaxies grouped together?**

- Spiral galaxies are often found in groups of galaxies (up to a few dozen galaxies).
- Elliptical galaxies are much more common in huge clusters of galaxies (hundreds to thousands of galaxies).

### 2 Galaxy formation

#### **How do we observe the life histories of galaxies?**

- Observing galaxies at different distances shows us how they age.

Deep observations show us very distant galaxies as they were much earlier in time (old light from young galaxies).

#### **How do we study galaxy formation?**

- We still can't directly observe the earliest galaxies.
- Our best models for galaxy formation assume:
  - Matter originally filled all of space almost uniformly.
  - Gravity of denser regions pulled in surrounding matter.

*Different way galaxies can form:*

- Denser regions contracted, forming **protogalactic clouds**.
- Hydrogen and helium gas in these clouds formed the first stars.
- Supernova explosions from the first stars kept much of the gas from forming stars.
- Leftover gas settled into a spinning disk due to the **conservation of angular momentum**.