Optimal Portfolio Choice

Portfolio Returns

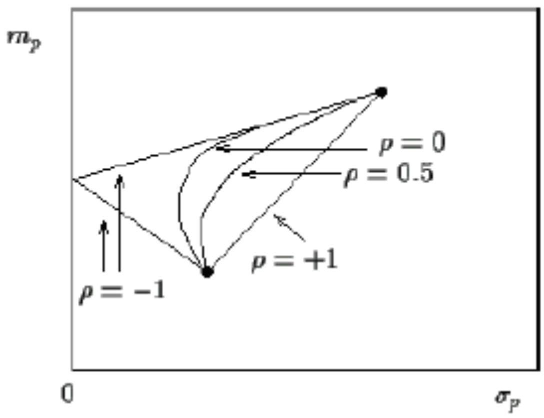
- $\tilde{r}_p(s) = \sum w_i \tilde{r}_i(s)$
  - $\tilde{r}_p$ – Unknown future return on portfolio
  - w/w_i – Is asset I's weight in the portfolio and is decided before we know *s*
    - w_i – Dollar amount invested in *i*
    - w – Dollar value of the portfolio
- $r_p = \sum w_i r_i$
  - Realised return on portfolio
- $E(\tilde{r}_p) = \sum w_i E(\tilde{r}_i)$

Covariance and Correlation

- **Covariance** – measures how much two variables $\tilde{r}_i$ and $\tilde{r}_j$ vary together
  - $\sigma_{ij} = E\left[(\tilde{r}_i - E(\tilde{r}_i))\left(\tilde{r}_j - E(\tilde{r}_j)\right)\right]$
    $= \sum p(s)[\tilde{r}_i - E(\tilde{r}_i)] \times [\tilde{r}_j - E(\tilde{r}_j)]$
  - Covariance of a variable with itself:
    - $\sigma_{ii} = \sigma_i^2$
  - If covariance between the returns of two assets is positive → portfolio s.d. will be less than the weighted average of individual assets' s.d. UNLESS the two assets are perfectly correlated.
- **Correlation** – measures the strength of the LINEAR relationship between two variables
  - $\rho_{ij} = \frac{\sigma_{ij}}{\sigma_i \times \sigma_j}, \quad -1 \le \rho_{ij} \le 1$
  - Dividing by $\sigma_i \times \sigma_j$ gets rid of the scale
  - Investors benefit from diversification if correlation coefficient is less than perfectly positive
- **Variance** – portfolio risk
  - $\sigma_c^2 = \sum\sum w_i w_j \sigma_{ij} = \sum\sum w_i w_j \rho_{ij} \sigma_i \sigma_j$
    $= w_a w_a \sigma_{aa} + w_a w_b \sigma_{ab} + w_b w_a \sigma_{ba} + w_b w_b \sigma_{bb}$

Diversification

- $\rho_{ij} = -1$ → Maximum diversification benefits: can make a risk-free portfolio
- $\rho_{ij} = +1$ → No diversification benefits
- $-1 < \rho_{ij} < 1$ → Diversification benefits improve as curve bends backward.
- **Lower the correlation** → smaller the variance of MV portfolio
- Number of individual variances in a portfolio = N
- Number of individual covariances = $\frac{N^2 - N}{2}$



Markowitz Portfolio Selection

- **Minimum-variance frontier** – lowest possible variance/risk for each expected returns
- **Global minimum variance portfolio** – portfolio with minimum possible variance out of all risky portfolios
- **Efficient Frontier** –Top –half of M.V. frontier
  - Rational investors will choose portfolio on efficient frontier
- **CAL** – results from combinations of market portfolio and risk-free asset
  - Combinations that move CAL in northwest direction are strictly preferred
- **CML** – best CAL
  - Tangent line from intercept point on efficient frontier to where expected return equals risk-free rate of return.
  - Points on CML are superior to efficient frontier → includes risk-free asset in portfolio.
- **Tangency portfolio** – portfolio with highest Sharpe ratio
  - If r_f is available and input lists are identical → all investors hole identical risky portfolio (Tangency portfolio)