University of Melbourne

FNCE 10002

PRINCIPLES OF FINANCE

LECTURE NOTES

TABLE OF CONTENTS

Topic 4: Introduction to Options

Probability Distribution Approach

- One assumes that investors can specify the possible outcomes (C1 through Cn) and associate probabilities or likelihoods (p1 through pn) with these outcomes).
- For each state, convert the cash flows into rate of returns using the initial investment of C.

State	*Probability*	*Cash Flows*	*Rate of Return*
1	p_1	C_1	$R_1 = (C_1 - C)/C$
2	p_2	C_2	$R_2 = (C_2 - C)/C$
3	p_3	C_3	$R_3 = (C_3 - C)/C$
:	:	:	:
n	p_n	C_n	$R_n = (C_n - C)/C$

- Note that p1 + p2 + … + pn = 1 because one of these states of the world will actually occur.
- The **EXPECTED RETURN** is the expected outcome measured as the weighted average of the individual outcomes, given by:

$$E(r) = p_1R_1 + p_2R_2 + \ldots + p_nR_n$$

- The **VARIANCE** or **STANDARD DEVIATION** of returns is the measure of dispersion around the expected return.
- The greater the dispersion, the higher the uncertainty and risk, given by:

$$\text{Var}(r) \text{ or } \sigma^2 = p_1[R_1 - E(r)]^2 + p_2[R_2 - E(r)]^2 + \ldots + p_n[R_n - E(r)]^2$$

$$SD(r) = \sigma$$

- Note that the variance and standard deviation take into account returns above and below the expected return.
- Investors are typically concerned with returns below the expected return (that is, **DOWNSIDE RISK**).

Example: The current price of Stock X is \$10 and market analysts expect the following three states to occur one year from now. What is the expected return and standard deviation of return for this investment?

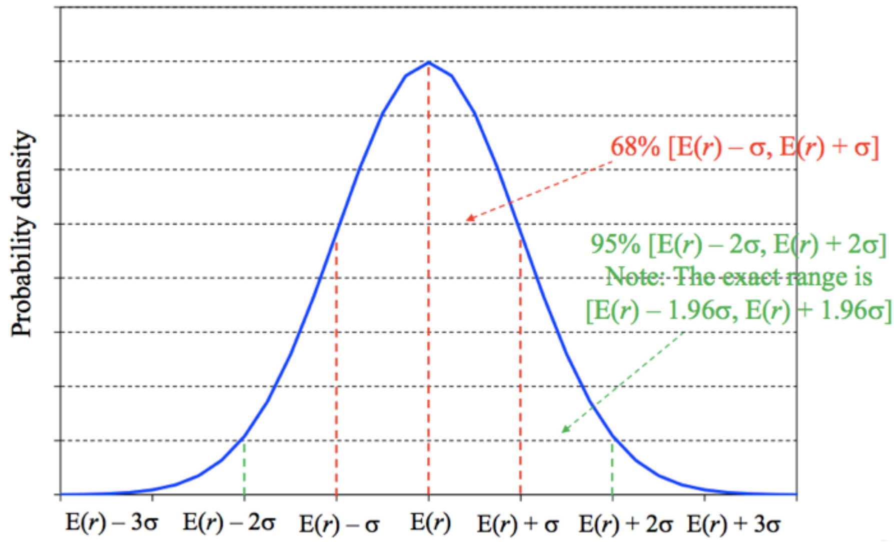
State of the Market	*Probability*	*Dividend*	*Price*
Sucks	0.3	\$0.50	\$8.50
Good	0.4	\$1.00	\$10.00
Awesome	0.3	\$1.50	\$10.50

- We first convert the cash flows into rates of return for each state
- The return distribution for Stock X is as follows

State	*Probability*	*Rate of Return, R*
Sucks	0.3	(0.50 + 8.50 – 10.00)/10.00 = –10.0%
Good	0.4	(1.00 + 10.00 – 10.00)/10.00 = 10.0%
Awesome	0.3	(1.50 + 10.50 – 10.00)/10.00 = 20.0%

- $E(r) = 0.3(-0.1) + 0.4(0.1) + 0.3(0.2) = 7.0\%$
- $\text{Var}(r) = 0.3(-0.1 - 0.07)^2 + 0.4(0.1 - 0.07)^2 + 0.3(0.2 - 0.07)^2$
- $\text{Var}(r) = 0.0141$
- $SD(r) = (0.0141)^{1/2} = 0.1187$ or 11.9%

- A more general interpretation of the expected return and standard deviation of return requires assuming returns are continuous and normally distributed.
- Properties of a **NORMAL DISTRIBUTION** include:
  1. It is a bell-shaped distribution and requires only the expected (or mean) return and standard deviation of return to fully describe it.
  2. It is symmetric around its mean.
  3. It implies an unlimited downside loss potential.



- In the previous example for stock X, $E(r) = 7\%$ and $\sigma = 11.9\%$
- Assume you purchased 100 shares of Stock X, investing $1,000
- The expected value of your investment next period is…
  - $1000[1 + E(r)] = 1000(1.07) = \$1{,}070$
- There is a 95% probability that the *realized* return will lie in the range $[E(r) - 2\sigma, E(r) + 2\sigma]$ or (–16.8%, 30.8%)
- So, there is a 95% probability that a $1,000 investment in this security will be worth between $1000(1 - 0.168) = \$832$ and $1000(1 + 0.308) = \$1{,}308$ next period

Portfolio Risk and Return

- Investors are assumed to be risk averse; hence, the higher the variance or standard deviation of returns the worse off the investor.
- A **RISK AVERSE** investor's objective is to minimise the risk of portfolio of investments, given a desired level of expected return or to maximise the expected return of portfolio of investments, given a desired level of risk.
- The simplest way to minimise risk is to diversify across different securities by forming a portfolio of securities.
- Portfolio risk falls as the number of securities in the portfolio increases, but portfolio risk cannot be entirely eliminated using this method.
- The risk that cannot be eliminated is called **SYSTEMATIC RISK**.
- A portfolio's **EXPECTED RETURN** is the weighted average of the expected returns of its component securities.
- Note that the weighs are percentages of the investor's original wealth invested in each security.
- It is assumed that all the available funds are invested in the two securities, given by:

$$E(r_p) = x_1 E(r_1) + x_2 E(r_2)$$

- x_j = Amount invested in security j / Total amount invested
- *Note:* $x_1 + x_2 = 1$ and $x_1 = 1 - x_2$ (or $x_2 = 1 - x_1$)

- A portfolio's **VARIANCE** is the weighed average of the variance of its component securities and the covariance between the securities' returns.

$$Var(r_p) \text{ or } \sigma_p^2 = x_1^2\sigma_1^2 + x_2^2\sigma_2^2 + 2x_1x_2\sigma_{12}$$

❖ σ_{12} *or* $Cov(r_1, r_2)$ = Covariance between securities 1 and 2

- The standard deviation of the portfolio is hence:

$$SD(r_p) \text{ or } \sigma_p = [x_1^2\sigma_1^2 + x_2^2\sigma_2^2 + 2x_1x_2\sigma_{12}]^{1/2}$$

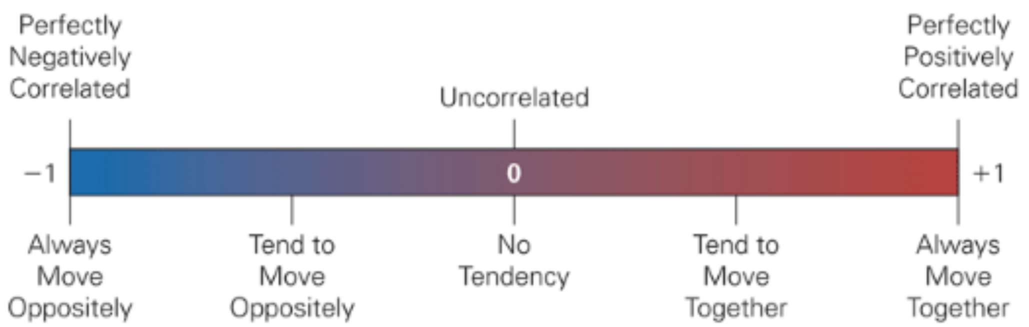
- The **COVARIANCE OF RETURNS** measures the level of complement between security returns.

$$\sigma_{12} = p_1[r_{11} - E(r_1)][r_{21} - E(r_2)] + \ldots + p_n[r_{1n} - E(r_1)][r_{2n} - E(r_2)]$$

r_{jk} = Return on security $j = 1, 2$ in state $k = 1, 2, \ldots, n$

- If $\sigma > 0$, then above (below) average returns on security 1 tend to coincidence with above (below) average returns on security 2.
- If $\sigma < 0$ then above (below) average returns on security 1 tend to coincide with below (above) average returns on security 2.
- If $\sigma = 0$ then security 1's return tends to move independently of security 2's return.
- Note that the magnitude of covariance changes depending on how returns are measured - percentages versus decimals.
- The **CORRELATION OF RETURNS** is a standardised measure of co-movement between two securities, given by: $\rho_{12} = \sigma_{12}/\sigma_1\sigma_2$

- Note that the sign of the return correlation is the same as the sign of the return covariance and that its value is between -1 and 1.



- The covariance of returns can be rewritten as: $\sigma_{12} = \sigma_1\sigma_2\rho_{12}$