MATH1011

APPLICATIONS OF CALCULUS COURSE NOTES

Contents

MATH1011 Applications of Calculus

- Establishes and reinforces the fundementals of calculus, illustrated with context and applications.
- Differential calculus in solving optimisation problems and integral calculus in measuring how a system accumulates over time.
- Topics include:
  - Fitting of data to various functions
  - Interpretation and manipulation of periodic functions
  - Evaluation of commonly occurring summations.
- Diff calc:
  - Extended to 2 variables
- Integration:
  - Integration by substitution and evaluation of integrals of infinite type.

Outcomes:

- Analyse practical problems using techniques from differential and integral calculus
- Fit as appropriate linear, exponential and periodic functions with experimental data
- Sketch generalised sinusoidal functions
- Use differential calculus to solve optimisation problems of 1 variable
- Calculate the partial derivatives of functions of 2 variables, solve optimisation problems of functions of 2 variables
- Calculate finite sums and use sigma notation where appropriate
- Evaluate definite integrals and use definite integrals in applications
- Determine when improper integrals of infinite type exist

Assessment:

65% exam

15% quiz 1 mark (better mark principle)

15% quiz 2 mark (better mark principle)

2.5% for each assignment (of 2)

Elementary functions

Linear functions:

Definition:

Linear functions are of the form:

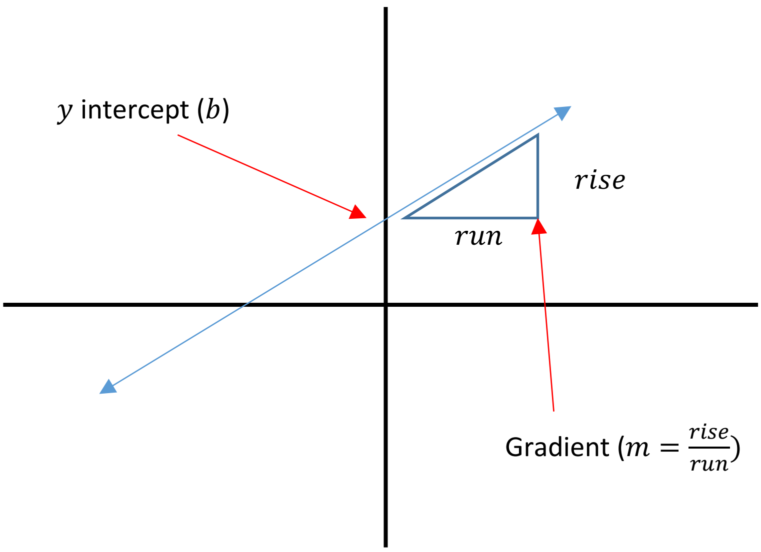
General form:

$$ax + by + c = 0$$

Gradient/intercept form:

$$y = mx + b$$

(where $m =$ the gradient of the line, and $b =$ the $y -$ intercept in the xy plane)

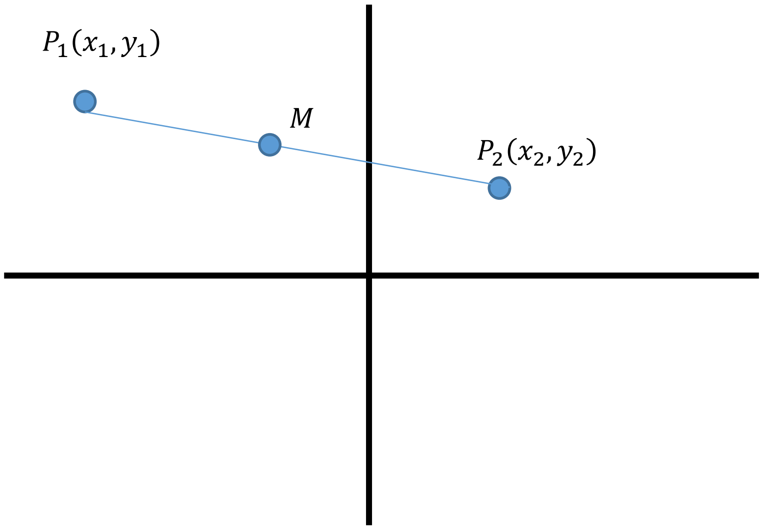


Gradient:

The **gradient** of a line is the lines ‘slope’ (can be positive, negative, zero or ‘infinite’)

- Positive gradient is increasing with increasing x
- Negative gradient is decreasing with decreasing x
- 0 gradient is horizontal
- Vertical lines have undefined gradient (∞)

Midpoint of an interval



The midpoint of 2 points, (x_1, y_1) and (x_2, y_2) is given by

$$\boxed{M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)}$$

Example:

Midpoint of $(5,3)$ and $(-2,1)$ is $M\left(\frac{5-2}{2}, \frac{3+1}{2}\right) = M\left(\frac{3}{2}, 2\right)$

Gradient between 2 points

The gradient between points is the difference in the y values over the difference in the x values (rise/run). This becomes

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Example:

Gradient of the line between $(5,3)$ and $(-2,1)$ is

$$\frac{1-3}{-2-5} = \frac{2}{7}$$

Distance between 2 points

Using Pythagoras:

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Equation of a line between 2 points:

1. Find the gradient

2. Using point gradient formula $\boxed{y - y_1 = m(x - x_1)}$
3. Simplify

Example:

Find the equation between $(1,1)$ and $(7,4)$

Gradient is: $m = \frac{7-1}{4-1} = 2$

$\therefore$ line is:

$$y - 1 = 2(x - 1)$$
$$y - 1 = 2x - 2$$
$$2x - y - 1 = 0$$

Quadratic functions:

Definition:

Quadratic functions are of the form

$$y = ax^2 + bx + c$$

- They form a parabola when graphed

Alternative vertex form:

Can also be expressed in the form:

$$y = a(x - k)^2 + h$$

Where $V(k, h)$ is the **vertex** of the parabola

- When graphing; this is the parabola $y = x^2$ shifted vertically h and horizontally k
  - If $h > 0$ shifted up
  - If $h < 0$ shifted down
  - If $k > 0$ shifted right
  - If $k < 0$ shifted left

Completing the square to change from standard into vertex form:

We can complete the square of the equation, to change the quadratic into the vertex form.

Example:

$$y = x^2 - 4x + 1$$