Coupon Bonds

Pricing a Fixed-Coupon Bond

- Generates a set of future cash flows
- Is like a portfolio of constituent zeros
- If we know the prices (and hence zero rates) of constituent zeros, we can price off the zero curve:

$$P_0 = \frac{C}{1+Z_{02}} + \frac{C}{(1+Z_{02})^2} + \frac{C}{(1+Z_{02})^3} + \cdots + + \frac{C}{\left(1+Z_{0,T-1}\right)^{T-1}} + \frac{C+Par}{(1+Z_{0T})^T}$$

- If a bond's price is worth more than its par value it is trading at a premium
- If a bond's price is less than its par value, the bond is trading at a discount
- The bond's price is what it is calculated as, because if it was anything else, there would be an arbitrage opportunity (if it was higher could sell the bond and buy 3 zeroes with the money and pocket the difference)

Yield-to-Maturity (YTM)

- The single discount rate that equates the bond price to the present value of all future cash flows of the bond
- Yield is the bond's internal rate of return
- Bond dealers annualize YTM using simple interest (if yield is 4% per half year they will quote it as 8% p.a)

$$P_0 = \frac{C}{ytm}\left[1 - \frac{1}{(1+ytm)^T}\right] + \frac{Par}{(1+ytm)^T}$$

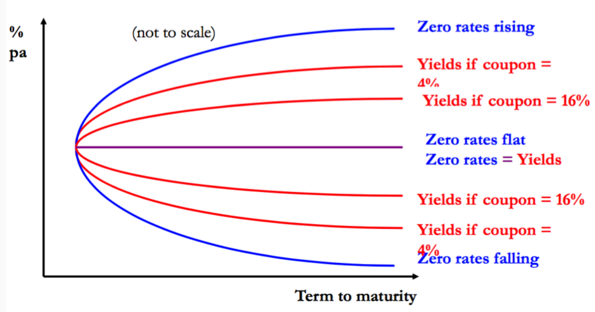
- Can think of yield as a complex kind of average of the interest rates of the constituent zeros, but the yield is also a function of the coupon rate

Differences Between Yields and Zero Rates

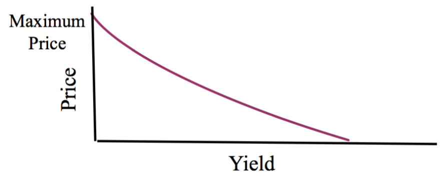
- Zero rate applies to every cash flow to be paid at that specific time and translates directly to prices
- A yield applies to that specific bond

The Yield Curve

- Plots YTM (vertical axis) for coupon bonds against their terms to maturity (horizontal axis)
- Typically close to, but is rarely coincident with, the zero rate curve.
- **Rising Zero Rates:** Yields are less than zero rates. Higher coupons decrease yields
- **Falling Zero Rates:** Yields are greater than zero rates. Higher coupons increase yields
- **All Zero Rates Equal:** Zero rates = Yields



Comparative Statics of Coupon Bonds



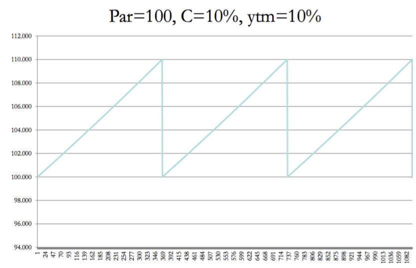
- If YTM increases, then P_0 decreases.
- If YTM decreases, then P_0 increases.
- A higher C produces a higher P_0.
- A lower C produces a lower P_0.

When price is calculated an instant after a coupon payment:

- If ytm < c, then P_0 > Par
- If ytm > c, then P_0 < Par
- If ytm = c, then P_0 = Par

Changing bond price after issue:

- There is a change in the required yield owing to:
  - Changes in the credit quality of the issuer
  - A change in the yield on comparable bonds (change in the yield required by the market)
- If the bond is sold at a premium or a discount, there is a change in the price of the bond, simply because the bond is moving toward maturity.
  - Discount bond price will increase toward maturity
  - Premium bond price will decrease toward maturity



RBA's Bond Pricing Formula

- Australian government bonds pay half-yearly coupons, so it is exactly half a year to the next coupon payment on only 2 days a year.
- h = number of days in the half-year ending on the next coupon payment
- f = number of days from the pricing date (0) to the next coupon payment date

$$P_0 = \frac{1}{(1+ytm_{hy})^{f/h}}\left[C + \frac{C}{ytm_{hy}}\left(1 - \frac{1}{(1+ytm_{hy})^{n-1}}\right) + \frac{Par}{(1+ytm_{hy})^{n-1}}\right]$$

Viewed from the payment date of the first coupon:

- it is an immediate cash flow of $*C* plus an **ordinary** annuity of *n* – 1 cash flows of $*C*, plus *Par. (annuity due)*

$$V_1 = C + \frac{C}{ytm_{hy}}\left(1 - \frac{1}{(1+ytm_{hy})^{n-1}}\right) + \frac{Par}{(1+ytm_{hy})^{n-1}}$$

$$P_0 = \frac{1}{(1+ytm_{hy})^{f/h}} \times V_1$$