Basic Probability

Assessing Probability

- **Probability:** chance/likelihood that an uncertain event will occur.
- *Impossible event = 0; certain event = 1.*
- **Event:** each possible outcome of a variable.
  - *Simple event* – single characteristic.
  - *Joint event* – two or more characteristics.
  - *Complement* of an event A (A') – all events not part of A (1-P(A)).

$$0 \leq P(A) \leq 1 \quad \text{For any event A}$$

- **Sample Space:** collection of all possible events.
- **Mutually exclusive events:** cannot occur simultaneously.
- **Collectively exhaustive events:** one of the events in the sample space must occur.
- **Approaches:**
  1. ***A priori:*** based on prior knowledge of the process

$$\text{probability an event occurs} = \frac{X}{T} = \frac{\text{number of ways the event can occur}}{\text{total number of outcomes}}$$

$$\text{probability of occurrence} = \frac{\text{number of ways the event has occurred}}{\text{number of trials}}$$

  2. ***Empirical:*** based on observed data (experimental probability)
  - empirical probability of joint event:

$$P(A \text{ and } B) = \frac{\text{number of outcomes satisfying } A \text{ and } B}{\text{total number of outcomes}}$$

  3. ***Subjective Probability:*** based on an individual's past experience, personal opinion and/or analysis of a situation.
- **General Addition Rule:**
  - Add together marginal probability of mutually exclusive events

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

- **Conditional Probability:** probability of one event, given that another event has occurred.

$$P(A \text{ and } B) = P(A|B) \times P(B)$$

$$P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)}$$ → The conditional probability of A given that B has occurred

$$P(B \mid A) = \frac{P(A \text{ and } B)}{P(A)}$$ → The conditional probability of B given that A has occurred

Where P(A and B) = joint probability of A and B
P(A) = marginal or simple probability of A
P(B) = marginal or simple probability of B

- **Independence:** 2 events are independent only if…
  - Independent when probability of one event is not effected by another event.
  - Conditional probabiltiy unchanged from marginal probability.

$$P(A \mid B) = P(A)$$

- **Multiplication rule:**

$$P(A \text{ and } B) = P(A \mid B)P(B)$$

- **Marginal Probability:**

$$P(A) = P(A \mid B_1)P(B_1) + P(A \mid B_2)P(B_2) + \cdots + P(A \mid B_k)P(B_k)$$

- Where $B_1, B_2, \ldots, B_k$ are k mutually exclusive and collectively exhaustive events

<u>Bayes Theorem</u>

- Used to revise existing probabilities (marginal) with new information.
- Extension of conditional probability – reverse conditioning between two events.

$$P(B \mid A) = \frac{P(A \mid B)P(B)}{P(A)}$$

- where:
  - B = event of interest
  - A = new event that might impact P(B)

NB $P(B \mid A) = \frac{P(A \text{ and } B)}{P(A)}$