ECON2102: MACROECONOMICS 2

WEEK 1: A MODEL OF PRODUCTION

MEASURING GDP

- GDP is defined as the market value of the final goods and services produced in an economy over a certain period
- There are three measures; the expenditure measure, the income measure and production measure
- Production = expenditure = income

1. The Expenditure Approach to GDP

- The national income identity: $Y = C + I + G + NX$
- Where Y is GDP
- This approach divides goods and services purchased into several categories

2. *The Income Approach*

- GDP is equal to the sum of all income earned in the economy – wages, salaries, operating surpluses of businesses, health and retirement benefits
- Can also assign all income to either labor or capital which refers to the inputs into production other than labor

3. *The Production Approach*

- Can be thought of as the value added approach as it is only the final goods and services that count towards GDP
- Computed by subtracting intermediate products

CHAPTER 3: AN OVERVIEW OF LONG RUN ECONOMIC GROWTH

ECONOMIC GROWTH

- The growth rate in some variable y is the percentage change in that variable

$$g = \frac{y_{t+1} - y_t}{y_t}$$

- The constant growth rule: $y_t = y_0(1 + \bar{g})^t$

$$\bar{g} = \left(\frac{y_t}{y_0}\right)^{1/t} - 1$$

THE RULE OF 70 AND THE RATIO SCALE

- If y_t grows at a constant rate g per year, then the number of years it takes y_t to double is approximately 70/g
- Small differences in growth rates result in large differences over time
- In a ratio scale, each interval represents a doubling so you squish the vertical axis
  - A variable growing at a constant rate appears as a straight line on a ratio scale

PROPERTIES OF GROWTH

- Use log laws

1. If $z = x/y$, then $g_z = g_x - g_y$.
2. If $z = x \times y$, then $g_z = g_x + g_y$.
3. If $z = x^a$, then $g_z = a \times g_x$.

CHAPTER 4: A MODEL OF PRODUCTION

SETTING UP THE MODEL

- A certain number of inputs are used in the production of the good
- Inputs: Labour (L) and Capital (K)
- The production function shows how much output (Y) can be produced given any number of inputs

$$Y = F(K, L) = \bar{A}K^{\frac{1}{3}}L^{\frac{2}{3}}$$

- $\bar{A}$ is a productivity parameter
- The production function exhibits constants returns to scale if doubling each input exactly doubles output
- If the exponents add to one, the function delivers constant returns to scale
- If doubling inputs more than doubles output, there is increasing returns to scale and the exponents add to more than one
- If doubling inputs less than doubles output there is decreasing returns to scale and the exponents add to less than one

ALLOCATING RESOURCES

- Want to figure out how many workers and machines to use by solving the following problem:

$$\max_{K,L} \pi = F(K, L) - rK - wL$$

- Where w is the wage rate and r is the rental rate of capital
- The solution is: MPK = r and MPL = w
- MPK is the extra amount of output produced when one unit of capital is added holding all other inputs constant
- MPL is the extra amount of output that is produced when one unit of labor is added, holding all other inputs constant
- Differentiate production function with respect to K and L respectively to solve

$$MPL = \frac{2}{3} \cdot \bar{A} \cdot \left(\frac{K}{L}\right)^{1/3} = \frac{2}{3} \cdot \frac{Y}{L}$$

$$MPK = \frac{1}{3} \cdot \bar{A} \cdot \left(\frac{L}{K}\right)^{2/3} = \frac{1}{3} \cdot \frac{Y}{K}$$

SOLVING THE MODEL

TABLE 4.1

The Production Model: 5 Equations and 5 Unknowns

Unknowns/endogenous variables: Y, K, L, r, w

Production function	$Y = \bar{A}K^{1/3}L^{2/3}$
Rule for hiring capital	$\frac{1}{3} \cdot \frac{Y}{K} = r$
Rule for hiring labor	$\frac{2}{3} \cdot \frac{Y}{L} = w$
Demand = supply for capital	$K = \bar{K}$
Demand = supply for labor	$L = \bar{L}$

Parameters/exogenous variables: $\bar{A}, \bar{K}, \bar{L}$

- A general equilibrium because more than one market is clearing
- A solution to the model is a new set of equations with the unknowns on the left side and the parameters and exogenous variables on the RHS
- The capital and labour markets clear when supply = demand which determines the wage and rental price of capital

FIGURE 4.2

Supply and Demand in the Capital and Labor Markets

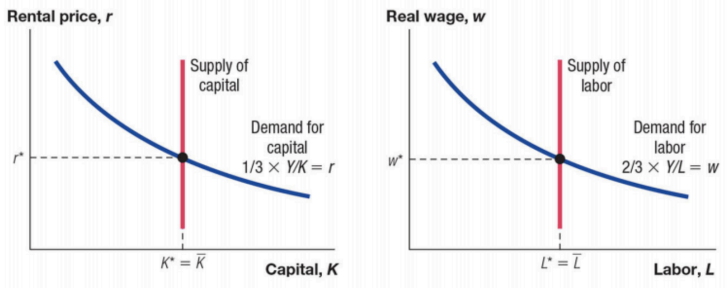

TABLE 4.2

The Solution of the Production Model

Capital	$K^* = \bar{K}$
Labor	$L^* = \bar{L}$
Rental rate	$r^* = \frac{1}{3} \cdot \frac{Y^*}{K^*} = \frac{1}{3} \cdot \bar{A} \cdot \left(\frac{\bar{L}}{\bar{K}}\right)^{2/3}$
Wage	$w^* = \frac{2}{3} \cdot \frac{Y^*}{L^*} = \frac{2}{3} \cdot \bar{A} \cdot \left(\frac{\bar{K}}{\bar{L}}\right)^{1/3}$
Output	$Y^* = \bar{A}\bar{K}^{1/3}\bar{L}^{2/3}$

INTERPRETING THE SOLUTION

- Learn that equilibrium wage is proportional to output per worker (Y/L)
- Equilibrium rental rate is proportional to output per capital (Y/K)
- Payments to capital and labour:

$$\frac{w^*L^*}{Y^*} = \frac{2}{3}, \text{ and } \frac{r^*K^*}{Y^*} = \frac{1}{3}$$

- Says that 2/3 of production is paid to labour and 1/3 is paid to capital
- This results in zero profits for the economy and verifies that production = expenditure = income

$$w^*L^* + r^*K^* = Y^*$$

ANALYZING THE PRODUCTION MODEL

- We need to solve for output per person as this determines a country's welfare
- Per capita = per person
- Per worker = per member of the labor force
- In our model, these two things are equal, so to solve for output per capita we can use L as L = population in our model
- Output per capita = y
- Capital per person = k = K/L

$$y^* \equiv \frac{Y^*}{L^*} = \frac{\bar{A}\bar{K}^{\frac{1}{3}}\bar{L}^{\frac{2}{3}}}{\bar{L}}$$
$$= \frac{\bar{A}\bar{K}^{\frac{1}{3}}}{\bar{L}^{\frac{1}{3}}}$$
$$y^* = \bar{A}\bar{k}^{\frac{1}{3}}$$

- This equation shows there are diminishing returns to capital and that output per person tends to be higher when the productivity parameter is higher and the amount of capital per person is higher
- When we assume $\bar{A} = 1$, the model predicts that most countries are substantially richer then they are

PRODUCTIVITY DIFFERENCES: IMPROVING THE FIT OF THE MODEL

- $\bar{A}$ measures how productive countries are at using their factor inputs to produce output → it is referred to as total factor productivity or TFP
- Assigning values less than one will improve our model
- We have no independent measure of this efficiency parameter so we make it the subject of the equation to calculate the level of TFP that would be needed to make our equation hold:

$$\bar{A} = \frac{y^*}{\bar{k}^{1/3}}$$

- A lower TFP implies that workers produce less output for any given level of capital per person
- Differences in capital per person explains about one quarter of the the difference in output between the richest and poorest companies
- TFP explains the remaining three quarters

UNDERSTANDING TFP DIFFERENCES

- Human capital is the stock of skills that individuals accumulate to make them more productive – workers in different countries possess different quantities of human capital
    - Can look at returns to education
- Technology – richer countries may use more modern and efficient technologies

- Institutions are in place to foster human capital and technological growth through property rights, the rule of law, government systems and contract enforcement
- Misallocation of resources means the industry's capital and labor is less productive then it should be

EVALUATING THE PRODUCTION MODEL

$$y^* = \bar{A}\bar{k}^{\frac{1}{3}}$$

- An implication of constant returns to scale is that output per person can be written as a function of capital per person
- However this relationship has diminishing returns – if we double the capital per person in a firm, the amount of output per person will be less than double

In the absence of TFP, the production model under predicts the differences in income