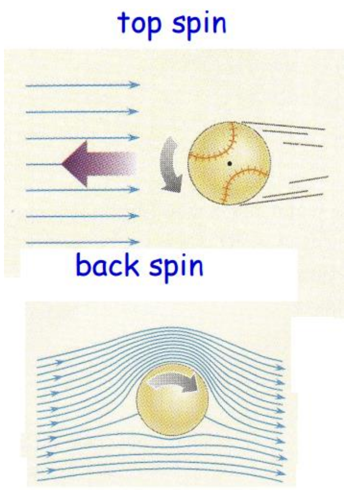
Applications of Bernoulli's principle

- Principle states that areas with faster moving fluids will experience less pressure

- Artery
    - When blood flows through narrower regions of arteries, the speed increases while pressure decreases
- Balls
    - When **top spin** is applied, the ball spins downwards creating **faster moving air on the bottom of the ball**. This faster moving air produces **lower pressure** that brings the ball down under the higher pressure from the top
    - When **back spin** is put on the ball, the **velocity of air on top of the ball increases**. This means that **pressure is lower on top of the ball**. This allows the ball to stay in the air for a longer time.

Remember that objects will have a "boundary layer" that encapsulates it.

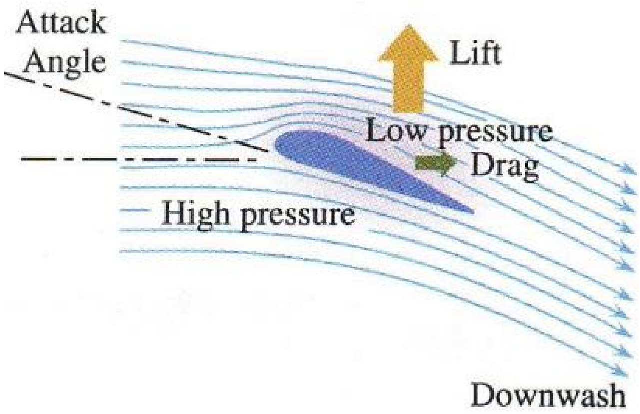


- Pitot tube
    - At the front of the pitot tube, the streamlines will separate and not enter.
    - On the sides, there are small holes that measure the pressure of the air that allows them to determine the speed
    - This then gives a reading of the plane's airspeed

- Chimney
    - Airstream above the chimney is faster and therefore has lower pressure. This means air leaves the chimney while air is prevented from moving into the higher pressure chimney.

- Aircraft Wing
    - On top the wing there is faster moving air and therefore lower pressure.
        - This contributes to lift

- **Lift:**
  - Bernoulli's Principle
    - Due to the differences in pressure. The higher pressure beneath the wing provides the aeroplane's lift



  - Newton's Principle
    - The "angle of attack" seen above determines how much lift is produced. Since air is pushed down as "downwash", there must be an equal and opposite reaction. This opposite reaction force provides the lift for the aeroplane.

- In the trail of planes, there are often circular wind patterns that form and can damage following aircraft
  - This is due to the differences in pressure
  - At the ends of the wings, the high pressure flows to the lower pressures (**air moves from high pressure to low pressure)** and produces turbulent air
  - This is reduced by constructing tall points on the end of wings that separate the different pressures and reduce the effect

- **Why does a plane stall?**
  - This occurs when there is not enough air moving over the wing to create the lift
    - Could possibly result from the "angle of attack" being to great

- **Condensation above the wings**
  - Air is full of water vapour.
  - Suddenly reducing the pressure and temperature can produce the fog seen above aircraft wings
    - <u>This change of pressure can occur during manoeuvres such as releasing the flaps or lowering air speed</u>
  - This is possible because air is compressible and can change temperatures

<u>Example</u>

A water pipe has a **diameter of 40mm** and enters a property from the street. At a distance of 5m from the street boundary it rises to the first floor of 3m. **It then reduces to a**

diameter of 15mm and runs to the tap. **If the flow rate from the tap is 1L/s**, what is the pressure difference between the water in the large pipe and the small pipe.

- Firstly, we need to recognise that they are at the same elevation. The information about the pipe rising is irrelevant because we are isolating one section of the pipe at the same elevation. Therefore, the potential energy part of the equation is the same and can be cancelled out.

$$p_2 + \frac{1}{2}\rho v_2^2 + \cancel{\rho g y_2} = p_1 + \frac{1}{2}\rho v_1^2 + \cancel{\rho g y_1}$$

- This leaves:
  - **$P_2 + ½pv_2^2 = P_1 + ½pv_1^2$**
- Rearranging this gives us:
  - **$P_1 - P_2 = ½p(V_2 - V_1)$**
  - **$P_1 - P_2 = ½p(\frac{Q^2}{A_2^2} - \frac{Q^2}{A_1^2})$**

Q = VA

V = Q/A

- We also know that **Q** (flow rate) **= V_1A_1** = V_2A_2

We are given **Q = 1L/s** but we need to convert this to m^3/s

- 1L/s > $0.001m^3/s$
- $A_1 = \pi(0.02)^2 = 0.00126m^2$
- $A_2 = \pi(0.0075)^2 = 0.000176m^2$
- Subbing them in gives a pressure difference of 15800Pa = 15.8 kPa

Non-ideal fluid <u>Viscosity</u>

- Viscosity causes drag as objects move through the fluid
- **To keep the fluid moving with constant speed, the pressure along the pipe drops**. This allows for Bernoulli's theory to remain correct
  - The size of this pressure difference depends on the viscosity
- The average velocity is given by the formula:

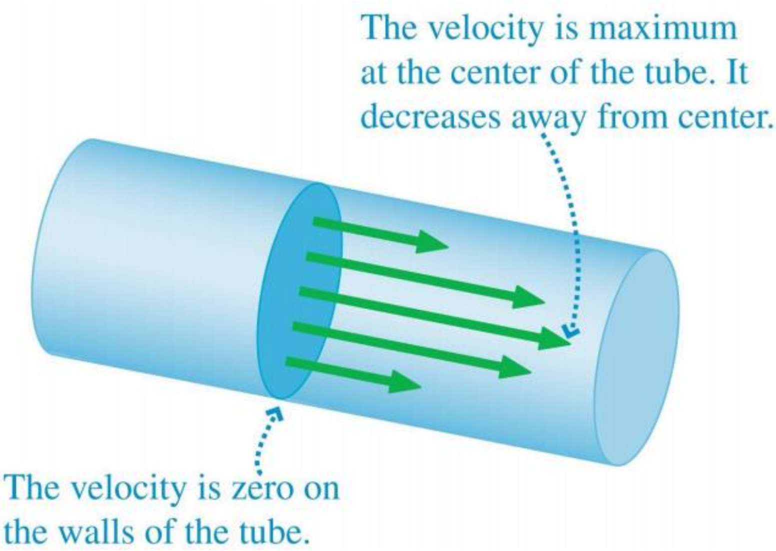
$$v_{avg} = \frac{A}{8\pi\eta}\frac{\Delta p}{L}$$

- **n stands for viscosity**, L for length of pipe, p is pressure difference
- **Units** are in **Pascals seconds** (Pa.s)
- <u>Example:</u>
  - An artery has a radius of 2.6mm and a length of 84mm. If the pressure drop is 400Pa, what is the average speed of blood? (n for blood is 4.0 x 10^{-3} Pa.s)

- Area = $\pi(0.0026)^2$ = $0.000021237m^2$
- **V** = $\frac{0.00000178392 \times 400}{8\pi(4.0x10^{-3})0.084}$ = 1m/s

Boundary Layer

- For real (viscous) liquids, the liquid at a boundary is stationary
- The **thickness** of the boundary **decreases with decreasing viscosity**



Poiseuille's Law

- This law relates pressure difference to the volume flow across a horizontal pipe
- **<u>It only applies to laminar flow</u>**
- Cannot be used for ideal fluids. They must have viscosity!! (n > 0)
- The formula for the average flow rate is given by:

$$Q = \pi R^2 v_{avg} = \frac{\pi R^4}{8\eta}\frac{\Delta p}{L}$$

- We can also find the resistance formula from this:

$$R \approx \frac{8\eta L}{\pi r^4}$$

Power dissipated during viscous flow

- Work needs to be done to keep a fluid flowing at a constant speed
  - This work ends up as heat energy

- The formula that relates the power and other equations is:

$$
\begin{aligned}
Power &= F_{net} \times v_{avg} \\
&= \Delta p A v_{avg} \\
&= \Delta p Q
\end{aligned}
$$

- The unit is the Watt (N.m/s or J/s)
- From the previous example, we can find the power dissipated

$$
\begin{aligned}
Power &= F_{net} \times v_{avg} \\
&= \Delta p A v_{avg} \\
&= 400 \times \pi (2.6 \times 10^{-3})^2 \times 1 = 8.5 \times 10^{-3} W
\end{aligned}
$$

<u>Example of all of these equations:</u>

When donating blood, a needle of diameter 1.2mm and length 15mm is used. The pressure at the exit of the needle is atmospheric while the gauge pressure of the vein is 8.0mmHg.

- What is the pressure difference in Pascals between the ends?
    - The difference is the gauge pressure = 8.0mmHg
    - Converting to Pascals: $8 \times 10^5 / 760$ Pa = 1052.6Pa

- What is the flow rate of blood through the needle?

$$Q = \pi R^2 v_{avg} = \frac{\pi R^4}{8\eta} \frac{\Delta p}{L} = \frac{\pi (0.6 \times 10^{-3})^4 \times 1052}{8 \times 4 \times 10^{-3} \times 15 \times 10^{-3}} = 8.91 \times 10^{-7} m^3 / s$$

- What is the average flow velocity of blood through the needle?

$$v_{avg} = \frac{Q}{\pi R^2} = \frac{8.91\times10^{-7}}{\pi(0.6\times10^{-3})^2} = 0.788 m/s$$

- How long will it take to collect 400mL of blood?

$$t = \frac{400\times10^{-6}}{Q} = 449 s$$

Reynolds Number

- Allows us to determine if fluid flow is laminar or Turbulent
- Acts as more of a "guide" rather than a rule
- **N < 2000 = Laminar**
- **N > 3000 = Turbulent**
- **2000 < N < 3000 = Unstable** (could be either)

$$N_R = \frac{\rho v_{ave} d}{\eta}$$

ρ density

v_{ave} average velocity

d diameter

η viscosity

- Example
  - An artery radius is 4mm, and the average flow speed is 2cm/s. Find the Reynolds number and determine if the flow is laminar or not. The density of blood is $1.1\times10^3 kg/m^3$ and its viscosity is $2.1\times10^{-3} Pa.s$.

$$N_R = \frac{\rho v_{ave} d}{\eta} = \frac{(1.1\times10^3 \text{kg.m}^{-3})(2.0\times10^{-2}\text{m.s}^{-1})(2\times4.0\times10^{-3}\text{m})}{2.1\times10^{-3}\text{Pa.s}}$$

$$= 84$$

SI Units!!

This is laminar flow since $N_R < 2000$.