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The Time Value of Money:

An Introduction to Financial Mathematics

Simple Interest: Interest paid only on the initial principal

$FV = F(1 + r_s n)$

$PV = F/(1 + r_s n)$

Compound Interest: Interest paid on principle and interest previously earned

$FV_n = F(1 + r)^n$

$PV = F/(1 + r)^n$

Annuities: A series of cash flows that are finite in number, evenly spaced and of *equal* value.

Ordinary Annuities: Time between now and the first cash flow is the same as the time separating each subsequent cash flow

$$Annuity\ PV\ Factor = \frac{1 - (1+r)^{-n}}{r}$$

$$FV\ of\ Annuity = P\left[\frac{(1+r)^n - 1}{r}\right]$$

$r = rate\ per\ period$

$n = number\ of\ periods$

$P = Periodic\ Payment$

$r = rate\ per\ period$

$n = number\ of\ periods$

Annuity Due: The first cash flow occurs immediately. However, the time between the first cash flow and the second cash flow is the same as the time separating all subsequent cash flows

$$P + P\left[\frac{1 - (1+r)^{-(n-1)}}{r}\right]$$

$$FV\ of\ Annuity\ Due = (1+r) \times P\left[\frac{(1+r)^n - 1}{r}\right]$$

$P = Periodic\ Payment$

$r = rate\ per\ period$

$n = number\ of\ periods$

$P = Periodic\ Payment$

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Deferred Annuity: The first cash flow occurs at some time in the future, but the time between now and the first cash flow does not equal the time separating each subsequent cash flow.

$$PV = \frac{F\left[\dfrac{1 - (1+r)^{-n}}{r}\right]}{(1+r)^{m-1}}$$

Perpetuities: A series of equally spaced cash flows of same dollar value that continues on forever.

Ordinary Perpetuity: The time between now and the first cash flow is the same as the time separating subsequent cash flows.

$$PV = \frac{F}{r}$$

Annuity Due Perpetuity: The first cash flow occurs immediately.

$$PV = F + \frac{F}{r}$$

Deferred Perpetuity: A perpetuity that commences some time in the future, but the time between now and the first cash flow does not equal the time separating each subsequent cash flow.

$$PV = \frac{\frac{F}{r}}{(1+r)^{m-1}}$$

Interest Rate Terminology

Annual effective interest rate: The frequency of charging / payment matches the period specified by the interest rate. In other words, the rate is quoted annually and is applied annually;

Annual nominal interest rate: Interest is charged more frequently than the time period specified in the interest rate. For example, 12% p.a. compounded monthly

Periodic interest rate: The rate of interest applied per compound period in the case of a nominal interest rate. For example, if the interest rate is 12% compounded semi-annually, the periodic interest rate is 12%/2 = 6%.