Topic 5: Game Theory

Making optimal decisions when it is no longer just you.

A Strategic Game consists of:

a) Players
b) Pure Strategies
c) Strategy Profile
d) Payoff / Utility Function

Strictly Dominant Strategy: A strategy that gives player i (strictly) higher payoff than any other of i's strategies, for any strategy of the opponent.
Strictly Dominated Strategy: A strategy that gives player i lower payoff than some other strategy sˆ of i, i for any strategy of the opponent.

Pure Strategy Nash Equilibrium
A strategy profile is PSNE is no player can gain by using some other strategy.

Ways to find PSNE:
(a) Try each strategy profile or star method
(b) Best responses meet each other

$$\text{(Home, home): } BR_A(home) = Home, BR_B(Home) = home \checkmark .$$

Mixed strategy: a vector of probabilities that tells us how often players will play each strategy. For example, $m_K = (0.6, 0.4)$. For example, Kicker and Goalkeeper game where there is no PSNE. There is a chance of playing left and right.

Mixed Strategy Nash Equilibrium

The following must hold:

$$u_K(L, \beta) = u_K(R, \beta).$$

		β	$1-\beta$
	K \ G	l	r
α	L	$p_L, -p_L$	$\pi_L, -\pi_L$
$1-\alpha$	R	$\pi_R, -\pi_R$	$p_R, -p_R$

$$u_K(L, \beta) = \beta u_K(L, L) + (1-\beta)u_K(L, R) = \beta p_L + (1-\beta)\pi_L$$
$$u_K(R, \beta) = \beta u_K(R, L) + (1-\beta)u_K(R, R) = \beta \pi_R + (1-\beta)p_R.$$

$$\beta^* p_L + (1-\beta^*)\pi_L = \beta^* \pi_R + (1-\beta^*)p_R$$
$$\Updownarrow$$
$$\beta^* = \frac{\pi_L - p_R}{\pi_R - p_R + \pi_L - p_L}.$$

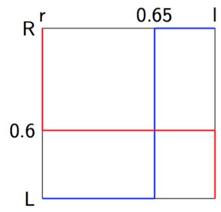
$$u_G(\alpha, L) = -\alpha p_L - (1-\alpha)\pi_R = -\alpha\pi_L - (1-\alpha)p_R = u_G(\alpha, R)$$
$$\Updownarrow$$
$$\alpha^* = \frac{\pi_R - p_R}{\pi_R - p_R + \pi_L - p_L}.$$

Therefore the mixed strategy NE is $((\frac{11}{15}, \frac{4}{15}), (\frac{4}{5}, \frac{1}{5}))$. This means that Player 1 must play strategy A *and* B with weights $(\frac{11}{15}, \frac{4}{15})$ for Player 2 to be indifferent between a *and* b. And Player 2 must play strategy a *and* b with weights $(\frac{4}{5}, \frac{1}{5})$ for Player 1 to be indifferent between A *and* B.

Topic 5B: Game Theory, Three Actions & Graphed Best Response

K \ G	l	r
L	0.63, -0.63	0.94, -0.94
R	0.9, -0.9	0.44, -0.44

$$\beta^* = \frac{\pi_L - p_R}{\pi_R - p_R + \pi_L - p_L},$$
$$\alpha^* = \frac{\pi_R - p_R}{\pi_R - p_R + \pi_L - p_L}.$$

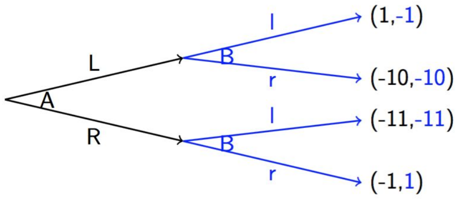


If G plays r with high probability, K wants to play L
If G plays ~~r~~l with prob. 0.65, K is indifferent
If G plays l with high probability, K wants to play R
If K plays R with high probability, G wants to play r
If K plays ~~R~~L with prob. 0.6, G is indifferent
If K plays L with high probability, G wants to play l

The equilibrium is where the two lines cross: K best responds to the G's strategy ~~(0.4,0.6)~~(0.6,0.4) by playing (0.65,0.35) (one of the mixed strategies K is indifferent among) and G best responds to (0.65,0.35) by playing ~~(0.4,0.6)~~(0.6,0.4).

Topic 6: Game Theory: Sequential Games

Strategy in sequential game is a complete contingent plan of action for each player. For this game, the strategy profile is (L,(l,r)) leads to payoffs (1,-1) and strategy profile (R,(l,r)) leads to (-1,1).



The matrix is different from simultaneous games:

A \ B	(l,l)	(l,r)	(r,l)	(r,r)
L	$1, -1$	$1, -1$	$-10, -10$	-10,-10
R	$-11, -11$	$-1, 1$	$-11, -11$	-1,1

Nash Equilibrium vs. Backward Induction

It could be that a strategy is NE but not solution for Backward Induction.

Subgame Perfect Nash Equilibirum

A strategy profile is a SPNE if it is a nash equilibrium in the overall game and in each of the subgames.
SPNE must be a NE, but not the other way around.

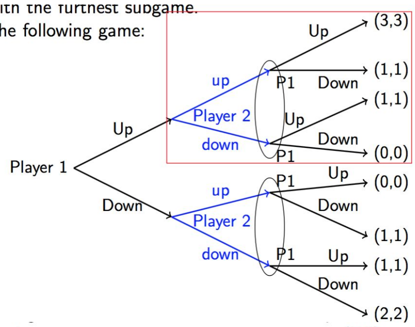
Information Sets

Spans the histories of a game that a player cannot distinguish. Sometimes, the subgame can become a simultaneous game when information sets is introduced. No IS in terminal nodes. Remember that in IS subgame, no need to put down three strategies.

Examples:
There is only one SPNE:

SPNE: ((Up,(Up,Down)), (up,down)) cause there is only one NE in both information set subgame.

we start with the furthest subgame.
Consider the following game:



There are two SPNE:

SPNE 1: ((Up,(Up,Up,Down,Down)), up)

SPNE 2: ((Down,(Up,Up,Down,Down)), down)

Cause there are two PSNE.

information sets.