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1. Introduction to Financial Mathematics

1.1. Simple versus Compound Interest

Simple interest is the value of a cash flow without any accrued interest to the principal.
eg/ a $1000 investment at 10% p.a. earning simple interest for 5 years would give you $500 (Future Value of $1,500)

Compound interest is the opposite. The interest is added to the principal and reinvested!
eg/ Same example as above…
FV at end of year 1 = 1,100
FV at end of year 2 = 1,100(1.1)= 1,210

1.2. Future and Present values

$$Firm\ Value\ (Present\ Value) = \frac{\sum E(CF_t)}{(1+r)^n}$$

$E(CF_t) = FV = Expected\ cash\ flows\ received\ at\ the\ end\ of\ period\ t$
$n = Number\ of\ periods\ over\ which\ cash\ flows\ are\ received$
$r = Rate\ of\ return\ required\ by\ investors$

Firm value is the ***present*** value of future expected cash flows.

Future Value of a Single Cash Flow (End of Period)
$F_n = P_0(1+r)^n$

eg/$1000 investment in 150 years at an interest rate of 6%
$F_{150} = 1000 * (1.06)^{150} = \$6{,}249{,}996$
Remember inflation.

Present Value of a Single Cash Flow (End of Period)
$P_0 = F_n(1+r)^{-n}$

eg/ You want to be a millionaire in ten years. How much would you need to save and invest if the interest rate is a pathetic 3%?
$P_0 = \frac{1000000}{(1.03)^{10}} = 744{,}102.99$
Good luck!

1.3. Unknown Interest Rates and Time Periods

Let's start with an example. This is one of those reversed problems.

eg/ You invest $1000 for a five year period. What interest do you need to double in that time period ($2000)? And at the interest rate of 10% how long will it take for these to triple in value?

First Case: $P_0 = \$1000, F_5 = 2000, n = 5$
$1000(1+r)^5 = 2000$
$r = 14.9\%$
Second Case: You know where it'll go. Only this time, we're solving for n.

1.4. Present value of perpetuities

What's perpetuity? It's an equal periodic cash flow that goes till eternity. (end of year CF)

$$P_0 = \frac{C}{1+r} + \frac{C}{(1+r)^2} + \cdots + \frac{1}{(1+r)^n}$$

$$So\ as\ n(period) approaches\ infinity, the\ formula\ above\ approaches \frac{1}{r}\ giving\ us$$

$$\mathbf{P_0} = \frac{\mathbf{C}}{\mathbf{r}}$$

A deferred perpetuity is the same thing as above but is deferred (CF starts at some future date) $P_0 = (\frac{C}{r})/(1+r)^n$ - Note n versus (n+1), n = deferred year-1!

Eg/ Three generous conglomerates decide to sponsor you for your brilliant scientific efforts. One guarantees you $100,000 a year forever with the first payment made at the end of year 1, Sponsor 2 guarantee $200,000 a year starting from the end of year 3 and Sponsor 3 wants to give $1.5 million upfront. What's the better deal? (Interest rate is 10% p.a.)

$$(Sponsor\ 1) P_0 = \frac{100{,}000}{0.10} = 1{,}000{,}000$$

$$(Sponsor\ 2, Upon\ deferral)\ P_0 = \frac{2{,}000{,}000}{(1.10)^2} = 1{,}652{,}892.56$$

From the looks of this, sponsor 2 has the best offer! 1,6 mil (2) > 1.5 mil (3) > 1 mil (1)

Growing Perpetuity

$$P_0 = \frac{C(1+g)}{1+r} + \frac{C(1+g)^2}{(1+r)^2} + \cdots + \frac{C(1+g)^n}{(1+r)^n}$$

$$g = constant\ rate\ of\ g\ percent\ per\ period, C(1+g) = \ CF$$

eg/ Dell offered a computer lease for an annual lease payment of $2000 next year with lease payments increasing at a constant annual rate of 3% forever. Assume an interest rate of 10% p.a. Ignore taxes and other complications. What happens if the first lease payment of $2000 is due immediately then grow at a constant annual rate of 3% p.a. forever?

The cash flow of $2000 implies that C(1+g)^0=$2000. PV = 2000/0.10-0.03 = 28,571.43

If the first payment is due immediately, PV =

No future values on this one. You have to be nuts to consider valuing an infinite period.

1.5. Present value of various types of annuity

Ordinary Annuities – series of equal periodic cash flows occurring at the end of each period lasting in a finite amount of period (n). (As apposed to perpetuity!)

$$P_0(OA) = (\frac{C}{r})(1 - (1+r)^{-n})$$

$$F_n(OA) = (\frac{C}{r})((1+r)^n - 1)$$

eg/

Deferred Ordinary Annuities

Annuities Due (CF on the beginning of each period)

End of period t is the same as the beginning of t+1. Present value will be bigger than that of an ordinary annuity ($$ value of time)

$$P_0(AD) = \left(\frac{C}{r}\right)(1 - (1+r)^{-n})(1+r)$$

$$F_n(AD) = (\frac{C}{r})((1+r)^n - 1)(1+r)$$

(1+r) = compounded once again due to the fact that CF is paid on the beginning of each period.

eg/ Amortization example…

1.6. **Effective Interest Rates**

When we want to know non-annual interest rates

$$r_e = \left(1 + \frac{r}{m}\right)^m - 1$$

r/m is the per period interest rate (and m is the no of periods in a year, quarterly: m = 4)

eg/

2. **Valuation of Debt Securities (also called bonds/debentures)**

2.1. **Characteristics of debt securities**

Debt securities are issued loans to help a company build capital. In return, the debt holder/purchasers (also known as discounters) gets a nice $$ return.

There are two types of debt instruments

- Short term debt instruments (matures within a year – Treasury Bill, Bank bills. Only pats the promised face value at maturity)
- Long term debt instruments (matures over a year)

Face (par) value is the amount paid at maturity (F_n or P_n)
Coupon rate is the interest rate (% of face value)
Interest (coupon) payment (C or I) is periodic payments made to debtholders
(coupon payment = coupon rate x face value)

underlined words belong to long term debt instruments only

eg/

A commercial paper with a face value of $100 and 90 days to maturity is issued at a yield of 5% per annum. What's the price of the security?

$$P = \frac{F}{1 + (r)\left(\frac{d}{365}\right)} = \frac{\$100}{1 + (.05)\left(\frac{90}{365}\right)} = \$98.78$$

It's being discounted because $100 will given back to you in 90 days (you lose bloody interest so the present value is less than its face value).

Now after 30 days of purchasing the commercial paper, you decide to sell the security and your friend agrees to buy it a 6% p.a. How much should he pay (no mate's rates)?

The paper now has 60 days left till maturity.

$$P = \frac{\$100}{1 + (0.06)(\frac{60}{365})} = \$99.02$$

More value because it'll be less time until you get your lump sum back.

2.2. **Valuation principle for financial securities**

THE PRICE OF A SECURITY TODAY IS THE PRESENT VALUE OF ALL FUTURE EXPECTED CASH FLOWS DISCOUNTED AT THE APPROPRIATE REQUIRED RATE OF RETURN (DISCOUNT RATE).