Week 4

Option Valuation: Chapter 21

Binomial Option Pricing Mode

Technique for pricing options based on the assumption that at the end of each period, the stock price has <u>only two possible values</u>

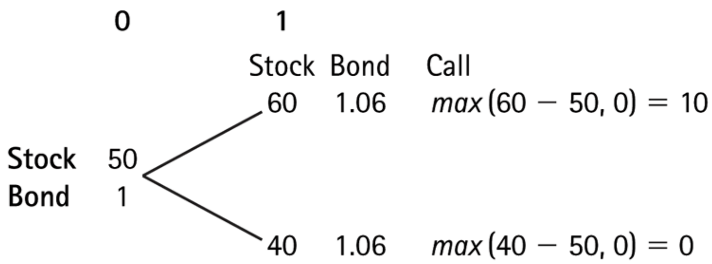
Binomial Tree

- A timeline with **two** branches at every date representing the possible events that could happen at those times

A Two-State Single-Period Model

Replicating Portfolio

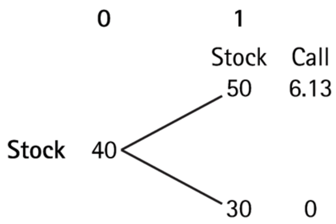
- A portfolio consisting of a **stock and a risk-free bond** that has the same value and payoffs in one period as an option written on the same stock
- *Law of One* implies that the current value of the call and the replicating portfolio must be equal
- ASSUME:
  - **European** call option expires in one period and has an exercise price of $50
  - The stock price today equals to $50 and the stock pays no dividends
  - In one period, the stock price either rise by $10 or fall by $10
  - The one-period risk-free rate is 6%



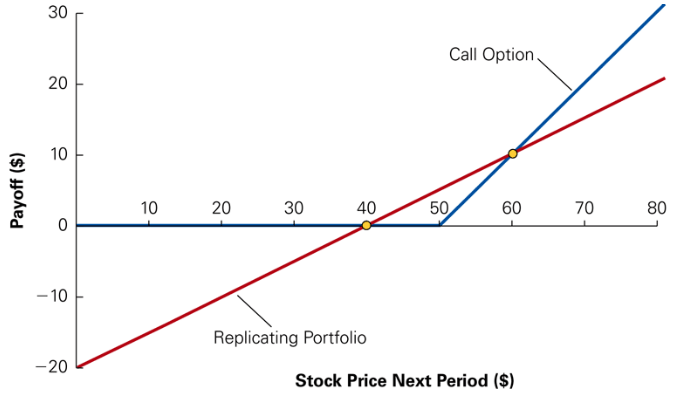
Eg:
D = Number of shares of stock purchased
B = Initial investment in bonds

- To create a call option using the stock and the bond, the value of the portfolio consisting of the stock and bond must match the value of the option in every possible state
  1. In the up state, value of the portfolio: $60\Delta + 1.06B = 10$
  2. In the down state, value of the portfolio: $40\Delta + 1.06B = 0$
  3. Using <u>simultaneous equations</u>, D & B can be solved for: D = 0.5, B = –18.8679
- A portfolio that is long 0.5 share of stock and short ~$18.87 worth of bonds will have a value in one period that exactly matches the value of the call:
  - 60 × 0.5 – 1.06 × 18.87 = 10
  - 40 × 0.5 – 1.06 × 18.87 = 0
- By the *Law of One Price*, the price of the **call option today must equal the current market value of the replication portfolio**
- The value of the portfolio today is the value of 0.5 shares at the current share price of $50, less the amount borrowed: $50D + B = 50(0.5) - 18.87 = 6.13$
- By using the *Law of One Price*, we are able to solve for the price of the option **without knowing theprobabilities of the states in the binomial tree**

❖ The value of the option today is therefore given by:



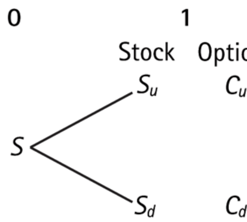
$$C = S\Delta + B$$



The Binomial Pricing Formula

❖ S = Current stock price
  - S will either go up to Su or go down to Sd next period
- ❖ Rf = Risk-free interest rate
- ❖ C = Value of call option
  - Cu if the stock goes up
  - Cd if the stock goes down

Given the above assumptions, the binomial tree would look like →



The Payoff of the replicating portfolios can be written as:

$$S_u \Delta 1 (1 + r_f)B = C_u \text{ and } S_d \Delta + (1 + r_f)B = C_d$$

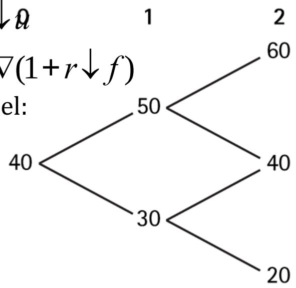
❖ Solving the two replicating portfolio equations for the two unknowns D & B yields the general formula for the replicating formula in the binomial model
  - Replicating Portfolio in the Binomial Model:

$$\nabla = C_u - C_d / S_u - S_d$$

$$B = C_d - S_d \nabla / (1 + r_f) \quad \text{OR} \quad B = C_u - S_u \nabla (1 + r_f)$$

  - Value of the Option: Option Price in the Binomial Model:

$$C = S\Delta + B$$

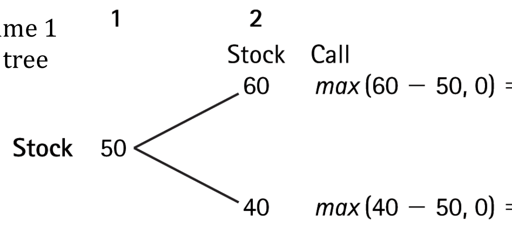


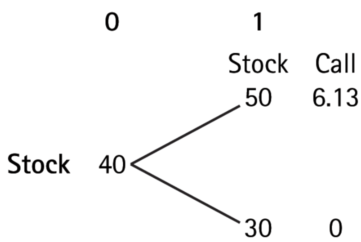
A Multi-period Model

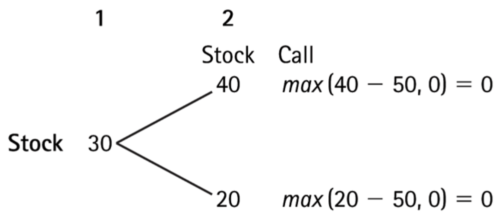
❖ Consider a two-period binomial tree for the stock price
  - Exercise price is $50 and one-period risk free rate is 6%

To calculate the value of an option in a multi-period binomial tree, start at the END of the tree and work backwards

1. At time 2, the option expires, so its value is equal to its intrinsic value
   - In this case: Call will be worth $10 if the stock price goes up to $60 and will be worth zero otherwise
2. To determine the value of the option in each possible state at time 1
   - If the stock price has gone up to $50 at time 1, the binomial tree will look like:
     (Value of option will be $6.13)







   - If the stock price has gone down to $30 at time 1. The binomial tree will look like: (Value of the option will be $0 since it is worth $0 in either state
3. To determine the value of the option in each possible state at time 0

Solving for D & B: $\Delta = \dfrac{C_u - C_d}{S_u - S_d} = \dfrac{6.13 - 0}{50 - 30} = 0.3065$ and

$$B = \frac{C_d - S_d\Delta}{1 + r_f} = \frac{0 - 30(0.3065)}{1.06} = -8.67$$

- Thus the initial option value is:

$$C = S\Delta + B = 40(0.3065) - 8.67 = \$3.59$$

A Dynamic Trading Strategy

- **Replication strategies** based on the idea that an **option payoff can be replicated by dynamically trading in a portfolio of the underlying stock and a risk-free bond**
- In the two-period above example, the replicating portfolio will need to be adjusted at the end of each period:
  - The portfolio price drops to \$30, the shares are worth \$9.20 and the debt has grown to \$9.20 (\$30 x 0.3065 = \$9.20 and \$8.67 x 1.06 = \$9.20)
  - The net value of the portfolio is worthless and the portfolio is liquidated
- If the stock price rises to \$50. the net value of the portfolio rises to \$6.13
  - The new D of the replicating portfolio is 0.5. Hence 0.1935 more shares must be purchased

  0.50 – 0.3065 = 0.1935
  - The purchase will be financed by additional borrowing

  0.1935 x \$50 = \$9.67
  - At the end of the total debt will be \$18.87

  This matches the value calculated previously

Making the Model Realistic

By <u>decreasing the length of each period</u>, and <u>increasing the number of periods</u> in the stock price tree, a realistic model for the stock price can be constructed.

The Black-Scholes Option Pricing Model

- A technique for pricing **European-style** options when the stock can be traded continuously.
- It can be derived from the Binomial Option Pricing Model by allowing the length of each period to shrink tozero and letting the number of period grow infinitely large
- Black-Scholes formula for puts is valid for European options, and the quotes are for American options

Black-Scholes Price of a Call Option on a Non-Dividend-Paying Stock

$$C = S \times N(d_1) - PV(K) \times N(d_2)$$

S = Current Price of stock

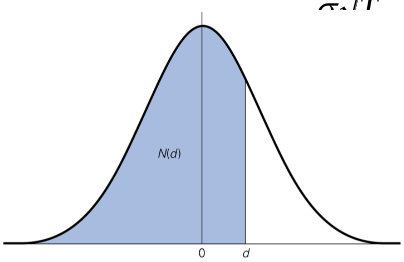
K = Exercise Price

N(d) = Cumulative normal distribution

- **Cumulative Normal Distribution**: Probability that an outcome from a standard normal distribution will be below a certain value

The Black-Scholes Formula:

$$d_1 = \frac{\ln[S / PV(K)]}{\sigma\sqrt{T}} + \frac{\sigma\sqrt{T}}{2} \quad and \quad d_2 = d_1 - \sigma\sqrt{T}$$



S = Annual Volatility

umber of years left to expiration

- N(d): Probability that a normal distributed random variable will take on a value less than d
- Probability = Area under the bell curve to the left of the point d

5 inputs are needed for the Black-Scholes formula:

1) Stock price
2) Strike Price
3) Exercise date
4) Risk-free rate
5) Volatility of the stock

European Put Options

- Using put-call parity, the value of a European put option
- Black-Scholes Price of a European Put Option on a Non-Dividend-Paying Stock:

$$P = PV(K)[1 - N(d_2)] - S[1 - N(d_1)]$$

Dividend-Paying Stocks

- If PV(Div) is the present value of any dividends paid prior to the expiration of the option, then:

$$S^x = S - PV(Div)$$

S^x = Price of the stock excluding any dividends

- Because a European call option is the right to buy the stock without these dividends, it can be evaluated by using the Black-Scholes formula with S^x in place of S
- A special case is when the stock will pay a dividend that is proportional to its stock price at the time the dividend is paid.
  - If q is the stock's (compounded) dividend yield until the expiration date, then:

$$S^x = S / (1 + q)$$

1. What is the key assumption of the **binomial option pricing model**?
   - There are only 2 possible prices for the underlying asset
   - 2 possible prices are the up-price and down-price
   - Underlying asset does NOT pay any dividends
   - Interest rate is constant throughout the life of the option
   - Investor are risk neutral
   - No taxes and transaction costs
 - Useful in valuing **AMERICAN** options
2. What are the key assumptions of the **Black-Scholes option pricing model**?
   - Options are **EUROPEAN** and can only be exercised at expiration
   - No dividends are paid out during the life of the option
   - No commissions
   - Risk-free rate and volatility of the underlying are known and constant
3. What is the **implied volatility** of a stock?
   - Implied volatility is the estimate of a stock's volatility which is **implied by an option's price**
   - The implied volatility from one option can be used to estimate the value of other options on the stock with the sae expiration date
   - Volatility is the only input not directly observable; practitioners use **two strategies to estimate the value of this variable:**
     1. Use historical data on daily stock returns to estimate the volatility of the stock over the past several months
     2. Use the current market prices of traded options

Week 5

Real Options: Chapter 22

Real Options

- The **right** to make a particular business decision (Eg Capital investment)
- Unlike financial options, real options are often not traded in competitive markets
- Underlying assets is physical rather than financial
- **DECISION TREE ANALYSIS:** branches of binomial trees represent uncertainty that CANNOT be controlled. Decision trees also include branches to represent different choices available to the decision maker
  - **Decision nodes:** choices available
  - **Information nodes:** Uncertainty is involved that is out of the control of the decision maker

Inputs for Black-Scholes Model

	Financial Option	Real Option
S	Stock Price	Market value of asset
X	Exercise Price	Upfront investment
σ	Stock return volatility	Asset value volatility
r	Risk free rate	Risk free rate
T	Time to maturity	Time to maturity
Div	Dividend	Free cash flow lost from delay

The Option to Delay an Investment Opportunity

Investment as a CALL OPTION (steps)

- **FCF = DIVIDEND PAID BY A STOCK**
  - Holder of call option does not receive the dividend until the option is exercised

1. Value of company at T=0

$$V_0 = \frac{FCF_1}{r - g}$$

2. NPV at T=0
3. NPV of waiting/delaying = **Black-Scholes Formula**
   - Find current value of asset without the dividend that will be missed; PV of lost cash flow uses project's **cost of capital**

     $$S^x = S - PV(Div)$$

   - Find PV of cost to start business at T=1: CF is certain so discount at **RISK-FREE RATE**

Factors Affecting the Time of Investment

- Without the option, it is optimal to invest as long as NPV >0 but **when you have the option of deciding to invest, it is optimal to invest only when the NPV is substantially greater than zero.**

- If we can always walk away from the project, the option of waiting will be positive; thus the NPV of investing today must be even higher for us to choose not to wait.

1) Volatility
- By delaying an investment, we can base our decision on additional information
- The **option to wait is most valuable** when there is a **great deal of uncertainty**

2) Dividends
- In real options, it is **ALWAYS better to wait unless there is a cost to doing so**(but never optimal to exercise call options early, absent dividends)
- The greater the cost, the less attractive the option to delay becomes
- By waiting before committing to an investment, a firm can obtain more information about the investment's returns
- Given the option to wait, an investment that currently has a negative NPV can be a positive value
- It is optimal to invest only when the NPV is substantially grater than 0

NPV EXAMPLE: DELAYING PROJECTS
- If project does not start now, investors have the **right, but not the obligation** to start the project in one year
  - European call option with strike price of (cost)
- If project starts in one year, investors **forego the FCF form first year**
- **FCF = Dividend paid by a stock**
  - Holder of call option does not receive any dividends until after the option is exercised
- **STOCK PRICE:**Current value of the project without the 'dividend' (FCF) that will be missed

$$S^x = S - PV(Div) = \$18,750,000 - \frac{\$1,500,000}{1.11} = \$17,398,649$$

- **EXERCISE PRICE:** PV of COST to begin project in one year

$$PV(X) = \frac{12,000,000}{1.04} = \$11,538,462$$

- Value of waiting one year to start project = Value of Call Option (computed from Black-Scholes model)
  - If NPV of starting project now > Value of waiting one year = It is optimal to begin project today

Growth Options

1. Growth Options
- A **real options to invest in the future** have value so they contribute to the value of any firm that has future possible investment opportunities
- Future growth opportunities can be thought of as a **collection of real call options** on potential projects
- Young firms tend to have higher returns than older, established firms

Steps in calculating value of option to proceed with project:
1) Find NPV of each outcome
   - NPV of investing today using PV of an annuity:

$$NPV = \frac{CF}{i}(1 - \frac{1}{(1+i)^n}) + CF$$

2) Find Weighted NPV of project (weighted by the probability of obtaining each outcome)

- **Weighted NPV = (% x NPV_1) + (% x NPV_2) = Value of option to proceed with project**

2. Option to expand

- Firms **may undertake projects in order to take on other projects in the future**
- In such cases, the initial project is an option that allows the firm to take other projects and the firm should be willing to pay a price for such options
- A firm may <u>accept a negative NPV on the initial project because of the possibility of high NPVs on future projects</u>

$$NPV = NPV_{without\ growth\ option} + PV_{growth\ option}$$

  - NPV of undertaking an investment is the **NPV initially calculated plus the value of the growth option we obtain by undertaking the project**
  - NPV → Rf
  - PV → (1 + Rf)
  - Even if project's current NPV is negative, NPV of investment opportunity can be positive and thus the firm should undertake it; it is **optimal to undertake the investment today ONLY because of the existence of the future expansion option**

Abandonment Options

Abandonment Options

- A **real option for an investor to cease making investments in a project**
- Abandonment options can **add value** to a project because a firm <u>can drop a project if it turns out to be unsuccessful</u>
- By exercising the option to abandon the venture, you limit your losses and the NPV of undertaking the investment becomes positive

RULES OF THUMB

- **Profitability index rule of thumb:**
  - Calls for investing whenever the profitability index *exceeds* some predetermined number
  - It is a way of accounting for the option to wait when there is **cash flow uncertainty**
- **Hurdle rate rule of thumb:**
  - Computes NPV using hurdle rate, a discount rate *higher* than the cost of capital (r), and specifies that the investment should be undertaken only when the NPV computed this way is positive
  - It is a way of accounting for the option to wait when there is **interest rate uncertainty**

QUESTIONS:

1. What is the difference between a real option and a financial option?
Real options give you the right to make a particular business decision whereas financial options give you the right to purchase or sell an asset at a predetermined price

2. What is the difference between an information node and a decision node on a decision tree?

3. What makes a real option valuable?
Real options are valuable if the NPV of undertaking the project is positive

4. Why can a firm with no ongoing projects, and investment opportunities that currently have negative NPVs, still be worth a positive amount?
By undergoing projects that currently have negative NPVs, NPV of investment opportunity could be positive, and thus the firm should undertake it and it is optima to undertake the investment today ONLY because of the existence of the future expansion option

5. Why is it sometimes optimal to invest in stages?
6. How can an abandonment option add value to a project?
    - ❖ An abandonment option can add value to a project because if the project ends up unsuccessful, you have the option to drop the project
7. Why is it inappropriate to simply pick the higher NPV project when comparing mutually exclusive investment opportunities with different lives?
8. How can you decide the order of investment in a staged investment decision?
9. Explain the **profitability index rule of thumb**.
   A regulation for evaluation whether to proceed with a project. Unlike NPVs, PI is a RATIO and ignores the scale of investment and provides no indication of the size of the actual cash flows. The rule states:
    - ➢ If **NPV > 0 = PI >1** = project is profitable and thus should proceed
    - ➢ If **NPV < 0 = PI <1** = Reject project
10. What is the **hurdle rate rule**, and what uncertainty does it reflect?
    The minimum rate of return on a project required by an investor. In order to compensate for risk, the riskier the project, the higher the hurdle rate
    - ➢ **If IRR > Hurdle rate = Accept project**
    - ➢ **If IRR < Hurdle rate = Reject project**

    The hurdle rate refers to the rate hedge fund managers must earn in their funds before they receive incentive-based compensation.