# Risk and Return

Computing Realised Returns

- T=0 – initial investment, T=1 = Ending Market Value and Dividends
    - Percentage Returns = the sum of the cash received and the change in the value of the asset, divided by the initial investment = (Dividend + Ending Market Value – Beginning Market Value) / Beginning Market Value

$$R_T = \frac{CF_1 + P_1 - P_0}{P_0}$$

$$\overline{R} = \frac{(R_1 + \cdots + R_T)}{T}$$ Or, conveniently written as $$E(R_{Asset}) = E(R) = \frac{\sum_{i=1}^{n} R_i}{n}$$

- **Average Returns** =
    - Average Return on Calculator = (1) clear stat – down shift and C STAT, (2) enter first value and (Sigma +) and the calculator will display n, number of items. (3) Continue adding values and pressing (Sigma +)
        - Mean of X and Y = Down Shift, 7 (x bar and y bar) for mean of x and for means of y press down shift and k (SWAP).
        - Sample Standard Deviation X and Y = Down Shift 8 (Sx,Sy) for X and Downshift 8(SWAP) for Y.
- **Holding Period Returns** – holding period returns apply if the stock is held for more than one year –


$$HPR = (1 + R_1) \times (1 + R_2) \times \cdots \times (1 + R_T) - 1$$

- **Risk Measures** – Variance

$$Var(R) = \frac{1}{T-1}[(R_1 - \overline{R})^2 + (R_2 - \overline{R})^2 + \ldots + (R_T - \overline{R})^2]$$

Or, conveniently written as

$$Var(R) = \sigma_R^2 = \frac{\sum_{i=1}^{n} [R_i - E(R)]^2}{n-1}$$

- Standard Deviation = $\sqrt{Var(R)}$

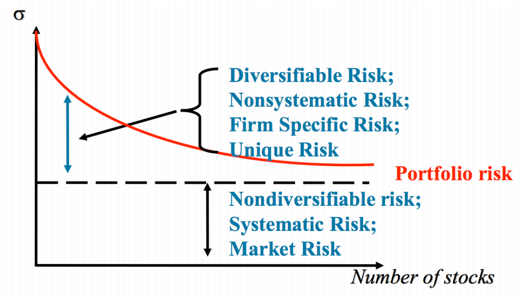
| Keys | Description |
|---|---|
| [↘] [C STAT] | Clear statistics memory. |
| *x*-value [Σ+] | Enter one-variable statistical data. |
| *x*-value [↘] [Σ-] | Delete one-variable statistical data. |
| *x*-value [INPUT] *y*-value [Σ+] | Enter two-variable statistical data. |
| *x*-value [INPUT] *y*-value [↘] [Σ-] | Delete two-variable statistical data. |
| [RCL] [Σ+] | Opens editor for reviewing or editing statistical data. |
| [↘] [7 $\bar{x},\bar{y}$] [↘] [SWAP] | Means of *x* and *y*. |
| [↘] [6 $\bar{x}_w$,b] [↘] [SWAP] | Mean of *x* weighted by *y*. Also calculates *b* coefficient. |
| [↘] [8 Sx,Sy] [↘] [SWAP] | Sample standard deviations of *x* and *y*. |
| [↘] [9 σx σy] [↘] [SWAP] | Population standard deviations of *x* and *y*. |
| [↘] [4 $\hat{x}$,r] [↘] [SWAP] | Estimation of *x*. Also calculates *r* correlation coefficient. |
| [↘] [5 $\hat{y}$,m] [↘] [SWAP] | Estimation of *y*. Also calculates slope and *m* coefficient. |
| [↗] [REGR 1] | Permits selection of six regression models or a best fit. Default is linear. |

- **Historical Returns** – Company stocks, provide a lower return but provides less risk than a small company's stocks which can provide a higher return but at a higher risk, unlike US Treasury Bills, provide a low return but also provide low risk.
- **Risk Premium**= is the added return (over and above the risk-free rate) resulting from bearing risk (also called excess return).
- **Types of Risk**

1. Firm Specific (Unsystematic) Risk – affects only a specific company and not other firms
2. Market (Systematic) Risk – affects the economy as a whole and affects all shares

**Diversification** – by investing in two or more assets whose values do not always move in the same direction at the same time, investors can reduce the risk of the portfolio – the risk that is diversified away in a portfolio is firm-specific and market risk is not diversifiable.

$$COV_{R_{1,2}} = \sum_{i=1}^{n} \left\{ p_i \times \left[ R_{1,i} - E(R_1) \right] \times \left[ R_{2,i} - E(R_2) \right] \right\}$$




- **Risk and Return Measures**

| | Risk and Return Measure | Risk and Return Measure with Probabilities |
|---|---|---|
| Average Return | $E(R_{Asset}) = E(R) = \frac{\sum_{i=1}^{n} R_i}{n}$ | $E(R_{Asset}) = \sum_{i=1}^{n} (p_i \times R_i)$ |
| Variance | $Var(R) = \sigma_R^2 = \frac{\sum_{i=1}^{n} [R_i - E(R)]^2}{n-1}$ | $VAR(R) = \sigma_R^2 = \sum_{i=1}^{n} \left\{ p_i \times [R_i - E(R_i)]^2 \right\}$ |
| Standard Deviation | $\sqrt{Var(R)}$ | $SD(R) = \sqrt{Var(R)}$ |

| **Covariance** | Written as on the Formula Sheet |
|---|---|
| $COV_{R_{1,2}} = \sum_{i=1}^{n} \left\{ p_i \times [R_{1,i} - E(R_1)] \times [R_{2,i} - E(R_2)] \right\}$ | $COV_{i,j} = \sum_{n=1}^{N} (R_i - \bar{R}_i) \times (R_j - \bar{R}_j) \times Pr_n$ |

**Correlation =** $\rho = \frac{\sigma_{R_{1,2}}}{\sigma_{R_1} \times \sigma_{R_2}}$

**Table 12-1 Statistics keys**

| Keys | Description |
|---|---|
| [shift] [C STAT] | Clear statistics memory. |
| *x*-value [Σ+] | Enter one-variable statistical data. |
| *x*-value [shift] [Σ+ / Σ-] | Delete one-variable statistical data. |
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| [shift] [5 / $\hat{y},m$] [shift] [SWAP] | Estimation of *y*. Also calculates slope and *m* coefficient. |
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**Portfolios** – the rate of return on the portfolio is a weighted average of the returns on the stocks and bonds in the portfolio:

$$r_P = w_B r_B + w_S r_S$$

- The expected rate of return on the portfolio is a weighted average of the expected returns on the securities in the portfolio:

$$E(r_P) = w_B E(r_B) + w_S E(r_S)$$

- Portfolio Correlation : Relationship depends on correlation coefficient If ρ = +1.0, no risk reduction is possible
  - If ρ = –1.0, complete risk reduction is possible
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- Diversification and Portfolio Risk - Diversification can substantially reduce the variability of returns without an equivalent reduction in expected returns, the reduction in risk arise because worse than expected returns from on asset are offset by better than expected returns from another.(Systematic portion of risk can be diversified away, therefore there is a minimum level of risk).
  - A systematic risk is any risk that affects a large number of assets, each to a greater or lesser degree.

- An <u>unsystematic risk</u> is a risk that specifically affects a single asset or small group of assets.

- **Total Risk = Systematic Risk + Unsystematic Risk**
    - Standard deviation of returns is a measure of total risk. For well diversified portfolios, unsystematic risk is very small.
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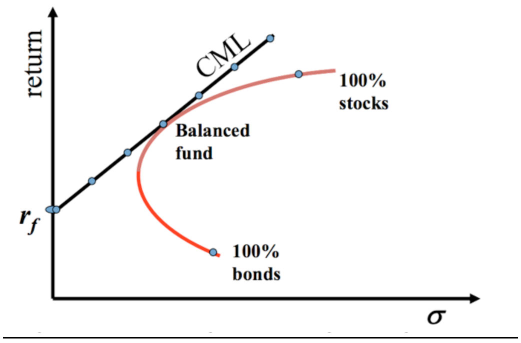


1. In a world that also has risk-free securities like T-bulls, investors can allocate their money across T-bills and a balanced mutual fund, with this risk-free assets available and the efficient frontier identified (we choose the efficient frontier identified, we choose the capital allocation line with the steepest slope- already identified above).
2. With the Capital allocation line identified – investors choose a point on the line – some combination of the risk-free asset and the market portfolio M, where they choose on the line depends on their risk tolerance.

**<u>Risk when Holding the Market Portfolio</u>** – best level of risk = Beta (b) of the security (measures the responsiveness of a security to movements in the market – systematic risk).

$$\beta_i = \frac{Cov(R_i, R_M)}{\sigma^2(R_M)} = \rho \frac{\sigma(R_i)}{\sigma(R_M)}$$

**<u>Relationship between Risk and Expected Return (CAPM)</u>** – expected return on an individual security (Capital Asset Pricing Model):

$$E(Ri) = R_F + \beta_i \times (\bar{R}_M - R_F)$$

| Expected return on a security | = | Risk-free rate | + | Beta of the security | × | Market risk premium |
|---|---|---|---|---|---|---|

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|---|---|
| **Expected Return on a Portfolio** | $E(R_{\text{Portfolio}}) = \sum_{i=1}^{n} [w_i \times E(R_i)]$ |
| **Expected Return of Two Assets** | $E(r_P) = w_1 E(r_1) + w_2 E(r_2)$ |
| **Variance of a Portfolio with Two Assets** | $VAR(Rp) = w_1^2 \sigma_{R_1}^2 + w_2^2 \sigma_{R_2}^2 + 2 w_1 w_2 \sigma_{R_{1,2}}$ |
| **Beta of a Portfolio with Two Assets** | $\beta_P = w_1 \beta_1 + w_2 \beta_1$ Or, generally: $\beta_P = \sum_{i=1}^{n} w_i \times \beta_i$ |