

## 1. Normal Distributions (Lecture 5)

The Normal Distribution

- Bell shaped and symmetrical about the mean
- Location is determined by the mean,  $\mu$
- Spread is determined by the standard deviation,  $\sigma$
- The random variable  $X$  has an infinite theoretical range from  $-\infty$  to  $+\infty$

Normal probabilities (using the standardised normal distribution –  $Z$ )

- The probability of a continuous variable having a specific value = 0
- Therefore, don't use greater than or equal to (or visa versa)
- Area to the left and right of  $\mu = 0.5$ , total area = 1

$$Z = \frac{X - \mu}{\sigma}$$

## 2. Sampling Distributions (Lecture 6)

Sampling Distribution Properties

- As  $n$  increases, standard deviation decreases (curve gets thinner and taller)

### 2.1 Sampling Distribution of the Sample Mean

$Z$  value for the Sampling Distribution of the Sample Mean:

$$Z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

$$\text{Standard Error} = \frac{\sigma}{\sqrt{n}}$$

Central Limit Theorem (CLT)

- If  $n$  is greater than or equal to 30, the sampling distribution of sample means will be approximately normally distributed (Only applies to  $\bar{x}$  questions)

### 2.2 Sampling Distribution of the Sample Proportion

$Z$  value for the Sampling Distribution of the Sample Mean:

$$Z = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}}$$

$$\text{Standard Error} = \sqrt{\frac{p(1-p)}{n}}$$

### 3. Confidence Intervals (I & II) (Lecture 7 and 8)

#### Confidence Interval

- Interval estimates, built around a sample statistic
- Draws conclusions from a sample statistic about a population parameter

#### Conclusion [sample]

- “Based on the sample mean, I have estimated, with 95% confidence, that the true population mean  $\mu$  will be between some upper and lower calculated value.”

#### Level of Confidence (LOC)

- X% of all confidence intervals, constructed using all possible samples, will contain the population parameter
- How confident you are that the confidence interval will contain the population parameter (will either contain it or not, not a probability)

#### General Confidence Interval Form

$$CI \text{ for LOC} = \text{Point Estimate} \pm (\text{critical value} \times \text{standard error})$$

#### 3.1 Population mean ( $\sigma$ known)

$$\mu: \bar{X} \pm Z_{crit} \times \frac{\sigma}{\sqrt{n}}$$

#### 3.2 Population mean (s known)

$$\mu: \bar{X} \pm t_{crit} \times \frac{s}{\sqrt{n}}$$

#### 3.2 Population proportion

$$p: \hat{p} \pm Z_{crit} \times \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}$$

#### 3.3 Determining the sample size for estimating the mean proportion ( $\sigma$ must be known)

$$n \geq \left( \frac{Z_{crit} \times \sigma}{E} \right)^2 \quad [\text{round up}]$$

#### Sampling Error (E) observations

- As sample size (n) increases, E decreases
- As spread of data ( $\sigma$ ) decreases, E decreases
- As LOC increases (Z), E increases

#### 3.4 Determining the sample size to estimate the true proportion

$$n \geq \frac{Z_{crit}^2 \times \hat{p}(1 - \hat{p})}{E^2}$$

- $\hat{p} = 0.5$  if there is no information at all given about the value of p or  $\hat{p}$ 
  - produces the maximum number of trials – never underestimates