

## Intertemporal Choice

### **Hyperbolic Discounting**

- People violate time consistency with regularity
- Beta-delta function
  - Utility  $U^0(u)$  of a utility stream  $u = (u_0, u_1, u_2, \dots)$  from the point of view of  $t = 0$  is
    - $U^0(u) = u_0 + \sum_{i=1}^{\infty} \beta \delta^i u_i$
  - When  $\beta = 1$ , an agent who discounts the future hyperbolically will behave exactly like an agent who discounts the future exponentially
  - Outcomes beyond the present time get discounted more than under exponential discounting
    - Exhibiting impulsivity

### **Choosing Not to Choose**

- Fear that buying in bulk may lead to overindulgence
- Issue is approached by drawing a distinction between naïve and sophisticated hyperbolic discounters
  - Self-control problems
    - A DM prefers  $x$  to  $y$  ahead of time, but  $y$  to  $x$  when the time arrives
  - Naïve time-inconsistent individuals are unaware of their self-control problems
  - Sophisticates are aware of their self-control problems
    - Make choices based on accurate predictions of future behaviour
- Layaway plans
  - Stores offering to hold onto savings for customers to ensure they don't spend, and can afford more expensive purchases
- Sophisticated consumers may exacerbate their own self-control problems
  - Tend to preoperate
    - Doing something now when it would be better to wait
    - Paradoxically results in situations when naïve individuals are better off than sophisticates

### **Preferences over Profiles**

- Preference for increasing utility profiles
  - When choosing between sequences of events, people will make a point of scheduling the unpleasant experience first and the pleasant one later
  - Could be captured by relaxing the assumption that  $\delta$  is less than one
    - If  $\delta > 1$ , a rational discounter will post-pone pleasant events as much as possible
      - Follows that  $\rho < 0$ , which is a negative time preference
    - Awkward solution, as it means that people will exhibit the same preferences in other contexts
- Preference for spread
  - People like to distribute multiple desirable events over time
- Preference for variation
  - Avoid choosing to consume the same good over and over again
- Preference over profiles
  - Individuals care about the shape of the utility stream as well as about individual utilities

- Peak-end rule
  - Used to assess the desirability of utility streams or 'episodes'
  - People consciously or unconsciously rank utility streams based on the average of the peak (the maximum utility) and the end (the utility near the end) and choose accordingly
  - Shape of the utility profile will be critically important
  - Entails duration neglect
    - Meaning that the length of an episode will be relatively unimportant
      - Contrary to exponential and hyperbolic discounting models

### **Misprediction and Miswanting**

- Underprediction of adaptation
  - People fail to appreciate the extent to which they will adapt to new conditions, such as a new endowment
  - People are unable to predict, ahead of time, just how attached they will be to an object after it has been incorporated in their endowment and loss aversion kicks in
  - Explains a no-questions-asked return policy
- Diversification bias
  - People overestimate the degree to which their future selves will enjoy variety over time
- Projection bias
  - People project their current preferences onto their future selves
- Hot-cold empathy gaps
  - Inability when in a 'hot' emotional state to empathise with people when in a 'cold' state and vice versa
  - When we are in a 'hot' state (experiencing hunger, thirst, anger, embarrassment or sexual arousal) we tend to underestimate how different our preferences are when we are in a cold state, and the other way around
- Miswanting
  - A mismatch between what we want because we think that we will like it when we get it and what we in fact like when we get it
  - Impact bias
    - Tendency to overestimate the enduring impact of future events on our emotional lives
    - Driven in part by underprediction of adaptation
      - Adapt to changing conditions to a much greater degree, and sooner, than they anticipate
    - Focussing illusion
      - Tendency for whatever you are attending to seem more important than it is

### **Applications**

- Implications of  $\beta - \delta$  preferences
  - O'Donoghue and Rabin (1999)
    - An agent will decide at  $t = 1$  whether or not to undertake an activity that has a once-off benefit ( $v$ ) and a one-off cost ( $c$ )
    - Immediate costs/investment good
      - Cost is incurred immediately at time of decision ( $t = 1$ )
      - Benefit  $v$  is enjoyed after a delay of one period ( $t = 2$ )
    - Immediate rewards/leisure good
      - Benefit is enjoyed immediately at time of decision

- Cost is incurred after a delay of one period
- If she does not do it, her outside option is 0 in both periods
- Ex-ante optimality (desired behaviour)
  - From the perspective of  $t = 0$  it would be desirable to undertake the activity at  $t = 1$  if
    - Immediate costs
      - $\beta[-\delta c + \delta^2 v] \geq 0$
      - Then  $\delta v \geq c$
    - Immediate rewards
      - $\beta[\delta v - \delta^2 c] \geq 0$
      - Then  $v \geq \delta c$
  - At  $t = 0$ , benefits and costs are both in the future, so both are affected equally by  $\beta$  which cancels out of the comparison
  - At  $t = 1$ , a time-consistent decision-maker ( $\beta = 1$ ) would behave according to these rules
- Actual behaviour
  - If she has a self-control problem ( $\beta < 1$ ), then at  $t = 1$  she chooses to actually undertake the activity if
    - Immediate costs
      - $-c + \beta \delta v \geq 0$
      - Then  $\beta \delta v \geq c$
      - Thus she undertakes too little of investment activities, because she discounts delayed reward more heavily than a time-consistent decision-maker does
    - Immediate rewards
      - $v - \beta \delta c \geq 0$
      - Then  $v \geq \beta \delta c$
      - Thus she undertakes too much of leisure activities, because she discounts delayed cost more heavily than a time-consistent decision-maker does
  - Consider the agent's expectations of her future behaviour, by letting  $b$  be her belief regarding the severity of her present bias
    - If  $\beta = b < 1$ , she is fully aware of her self-control problem
      - Fully sophisticated
    - If  $\beta < b = 1$  she is unaware she has a self-control problem
      - Believes she will make future decision in a time-consistent manner
      - Fully naïve
    - If  $\beta < b < 1$  she is aware she has a self-control problem, but underestimates how bad it is
      - Partially naïve
- Example
  - Cinema offers a mediocre movie ( $v = 3$ ) in week 1, a good movie ( $v = 5$ ) in week 2, a great movie ( $v = 8$ ) in week 3 and a Jonny Depp movie ( $v = 13$ ) in week 4

- Have an essay due in four weeks, and need to miss one of the movies
- Assume that  $\delta = 1$  and  $\beta = \frac{1}{2}$  and the benefit is the same regardless of which movie you miss
- For a time-consistent DM with  $\beta = 1$ , it is clearly least costly to miss the mediocre movie
  - She will write the essay in week 1
- For a present-biased DM with  $\beta = 1/2$ , the perceived cost of doing is later is discounted by  $\frac{1}{2}$ , making it tempting to delay
  - Costs of delaying writing essay when viewed from today (measured at different points)

		Skip movie in week			
		1	2	3	4
Discounted cost viewed from week	1	-3	$-\frac{1}{2}5$	$-\frac{1}{2}8$	$-\frac{1}{2}13$
	2		-5	$-\frac{1}{2}8$	$-\frac{1}{2}13$
	3			-8	$-\frac{1}{2}13$
	4				-13

- If fully naïve, she behaves myopically in each period, assuming that her future self has the same preferences as her present self
  - If she does not do it today, she believes she will do it at the time that is best, viewed through her current preferences
  - In week 1, she prefers to write the essay at 2 because  $3 > \frac{1}{2}5$ 
    - However in week 2, she prefers to do it in week 3 because  $5 > \frac{1}{2}8$
    - At week 4 she has no choice but to write the essay, tragically missing the Depp movie
  - This is naïve because her expectations of her own future behaviour are proven wrong
- If fully sophisticated, she chooses according to her current preferences, but correctly anticipates whether she would do the task at each future period, assuming she has not already done it
  - Realises that she plays a psychological game against her future selves, and the game is solved by backward induction
    - If not already done, she will do it at 4 when she has no choice
    - Knowing this, she would not do it in week 3 because  $8 > \frac{1}{2}13$
    - At 2 she correctly anticipates that if she does not do it now, it will not get done until 4
      - Knowing this she would do it at 2, given that  $5 > \frac{1}{2}13$
      - Knowing this, she would not do it in week 1 because  $3 > \frac{1}{2}5$
  - Sophisticate completes the task sooner than a naïf but later than a time-consistent DM
- Now suppose the schedule is the same as before, but instead of choosing one movie to skip, you can only see one of them
  - Seeing a movie thus has immediate benefits and delayed costs

- A time consistent DM holds out to week 4

		See movie in week			
		1	2	3	4
Discounted benefit viewed from week	1	3	$\frac{1}{2}5$	$\frac{1}{2}8$	$\frac{1}{2}13$
	2		5	$\frac{1}{2}8$	$\frac{1}{2}13$
	3			8	$\frac{1}{2}13$
	4				13

- Fully naïve DM
  - Viewed from week 1 it is best to wait for week 4
    - Same in week 2
    - In week 3, the immediate value of the week 3 movie outweighs the delayed value of the one in week 4
      - Caves into temptation in week 3
  - Naïve belief
    - In weeks 1 and 2 believed that she would indeed wait until week 4 when in reality she did not
- Fully sophisticated
  - The sophisticate knows that she will cave in in week 3
    - Then, the opportunity cost of seeing the good movie in week 2 is not the week 4 movie, but merely the great movie in week 3
    - Will then give in to temptation in week 2
      - Same in week 1
  - Thus chooses to see the movie in week 1
  - Sophistication helps to overcome the self-control problem in the case of immediate costs, but actually makes it worse in the case of immediate rewards
    - Knowing about future self-control problems can lead you to give in to them today, because you realise you will give in to them tomorrow
  - Indefinite procrastination – O'Donoghue and Rabin (2001)
    - Extend the model to investment type situations in which
      - There are an indefinite number of periods in which the agent can undertake the task, and she can do it at most once
      - There is a one-off immediate cost  $c$  when the task is completed
      - There is an infinite stream of benefits  $v$  that are enjoyed in every period, starting one period after the task is completed
        - Present value of this stream is  $\frac{\delta v}{1-\delta}$
    - Time consistent
      - Task is worth doing now if:
        - $-c + \frac{\delta v}{1-\delta} \geq 0$
      - For this agent, if it worth doing at all then it is best to do it right away, rather than at any other time in the future
    - If she is present biased

- $\beta < 1$  distorts her preference between doing it now as opposed to at some other point in future
- She will only do it now rather than in  $t$  periods time if
  - $-c + \beta \left( \frac{\delta v}{1-\delta} \right) \geq \beta \delta^t \left[ -c + \frac{\delta v}{1-\delta} \right]$
  - $\beta < 1$  imposes a heavier discount on delayed rewards on the left, making it less attractive to do it today
    - Waiting shrinks the term in square brackets on the right, which makes it desirable to do it soon
- If she is sophisticated, there is some maximum acceptable  $t$  past which she would prefer to simply do it now rather than wait
  - The more present biased she is, the longer the maximum delay
- She will invest today rather than wait until  $t$  if
  - $t \sim > \frac{1-\beta}{\beta} \cdot \frac{c}{v}$
  - This is maximum tolerable delay
    - It is zero for  $\beta = 1$  and goes to infinity as  $\beta$  goes to zero
    - It is also increasing in the up-front cost, and decreasing in the delayed benefit
- If the DM is at least partially naïve,  $\beta < b < 1$ , she believes that her maximum acceptable delay is shorter than it really is
  - She thinks that if she does not do it today, she will get around to it sooner than she actually will
  - This can lead her to continually put off the task, thinking that if she does not do it now she will get around to it soon enough
    - Could potentially procrastinate indefinitely
- Present biased but sophisticated DM correctly anticipates how she actually behaves in the future
  - She may use a commitment device that constrains her future actions to be more in line with her ex-ante wishes
    - Idea is to make it costlier to not keep to not keep to the ex ante plan
  - A time-consistent agent would never constrain her own future behaviour in this way
    - In the standard model, it is not possible to make yourself better off by imposing more constraints upon yourself
- Self-imposed deadlines, Ariely and Wertenbroch
  - If she is present-biased and at least partly sophisticated, this could act a commitment device to overcome procrastination
    - However if she is less than fully sophisticated, she might still not set the deadlines optimally
  - Experiment 2
    - Recruited 'native English speakers to held up proofread papers by other students to evaluate writing skills'
    - Generated meaningless essays and introduced grammatical and spelling errors

- Paid subjects for each error detected, but penalised subjects for each day late
  - Three groups
    - A: Three fixed, evenly spaced deadlines, each seven days
    - B: Self-imposed, binding deadlines, within a 3 week window
    - C: No deadlines – all three texts due after 21 days
  - For maximum flexibility, a time-consistent subject should set all three deadlines for the very last possible date
  - Model predicts performance across treatments as follows
    - Time consistent agent;  $B = C > A$
    - Full sophisticate with a self-control problem:  $B > A > C$ 
      - Will set the deadlines optimally
    - For fully naïve agent,  $A > B = C$
    - For partially sophisticated agent;  $A > B > C$ 
      - Will not set the deadlines optimally
- Findings
  - Found that evenly spaced deadlines are better than the deadlines that people put on themselves
    - Even when consumers demand a commitment device, they do not use it optimally
  - However self-imposed deadlines still better than a single, final deadline
- Self-Control at Work, Kaur, Kremer and Mullainathan (2015)
  - Workers are paid by piece rate, in a weekly pay check
    - Work is an activity with immediate costs and delayed benefits
  - Commitment contract
    - If the day's output falls below a target  $X$ , then the piece rate is halved to  $\frac{b}{2}$
    - For given output, this is dominated by the control contract
      - Direct effect of contract makes the worker worse off
    - Indirect effect via incentives
      - Penalty for low output makes it less attractive for the future self to slack off
      - If the effect on effort is big enough, this could make the worker better off in ex ante terms
    - However a worker would only choose this contract if she were sophisticated
  - if piece-rate workers are present-biased, they might work less hard than they themselves would consider ex ante optimal
    - Would also put in less effort when the pay day is distant, and more when it is nearer
    - Sophisticate may use a commitment device to encourage her future self to work harder
  - Design of experiment
    - Data entry firm
    - Workers are randomised to pay day
    - For each work day, each worker is randomised to one of four contract treatments, which they are told of the previous evening
      - Control
        - Fixed piece rate of  $b$  for each unit of output

- Target
  - Commitment contract is enforced, with one of three exogenously imposed targets
- Evening choice
  - Worker sets own target on previous evening
- Morning choice
  - Worker sets own target at start of next day
- Results
  - Workers choose nonzero targets on 35% of all opportunities
    - But 16% of workers always set a target of zero
  - Being in a choice treatment increases output by an average of 2% relative to control
  - Workers who show strongest payday effects are more likely targets, set higher targets and achieve larger output gains
  - Effects are persistent over time
  - Payday effects
    - Workers appear to work harder the closer to payday it is
    - Shows present biasedness independent of sophistication

### DellaVigna, 'Self-control problems'

- Laboratory experiments
  - Evidence suggests that discounting is steeper in the immediate future than in the further future
    - Induce time inconsistency
- Model
  - Quasi-hyperbolic model
    - $U^0(u) = u_0 + \sum_{i=1}^{\infty} \beta \delta^i u_i$
  - O'Donoghue and Rabin (2001) allow the agent to be partially naïve about the future self-control problems
    - Expects in the future period  $t + s$  to have the utility function
      - $\hat{U}_{t+s} = u_{t+s} + \hat{\beta} \delta u_{t+s+1} + \hat{\beta} \delta^2 u_{t+s+2} + \dots$
      - With  $\hat{\beta} > \beta$
    - Sophistication when  $\hat{\beta} = \beta$
    - Fully naïve when  $\hat{\beta} = 1$
  - If the agent could set consumption one period in advance, at  $t = 0$ , she would consume if  $\beta \delta b_1 + \beta \delta^2 b_2 \geq 0$  or
    - $b_1 + \delta b_2 \geq 0$ 
      - Where  $b_1$  is a consumption good and  $b_2$  is an investment good
    - However, the agent actually consumes at  $t = 1$  if
      - $b_1 + \beta \delta b_2 \geq 0$
      - Therefore consumes too little of the investment good and too much of the leisure consumption good
    - Agent expects to consume:
      - $b_1 + \hat{\beta} \delta b_2 \geq 0$
      - Overestimates the consumption of the investment good and underestimates the consumption of the leisure good
- Exercise
  - DellaVigna and Malmendier (2006)



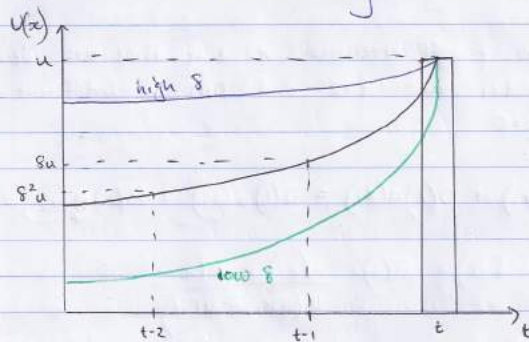
- Observe attendance at a gym where the monthly fee is \$80 per month, and an individual visit is \$10
  - Find that the users with a monthly contract attend only 4.4 times per month
  - Model with partially naïve members suggest two explanations for this
    - Users may be purchasing a commitment device to exercise more
    - May be overestimating their future health club attendance
- Homework and deadlines
  - Ariely and Wertenbroch (2002)
    - 51 professionals enrolled in a semester allowed to set their own binding deadlines for homework
    - According to the standard model, they should set deadlines for the last day of the semester
      - No benefit in setting early deadlines as they do not receive feedback
    - 68% of the deadlines are set for weeks prior to the last week
      - Indicates a demand for commitment
  - Ran a follow up experiment with three groups
    - Found that self-set deadlines improve performance over control group
      - Control group do not have any deadlines
    - However, deadline setting is not optimal
      - Group with equal-spaced deadlines does significantly better than the other groups

**Beshears, Choi, Harris, laibson, Madrian, Sakong; 'Self Control and Commitment: Can decreasing the liquidity of a savings account increase deposits?'**

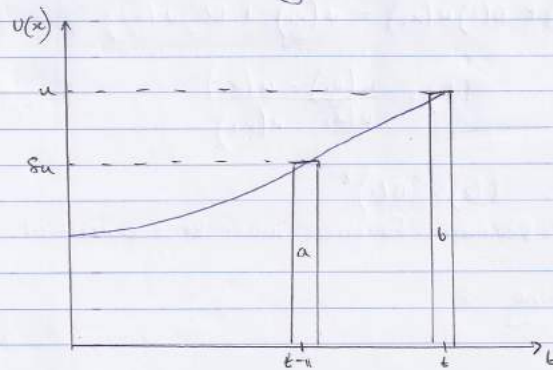
- Participant recruitment
  - Participants selected from the RAND American Life Panel, a panel of adults who are broadly representative of the US adult population
  - No overlap between participants in experiments
- Experiment 1
  - Participants randomly allocated a liquid account that they could withdraw from
    - Earned 22% annual interest rate
  - Also allocated an illiquid account with varying degrees of withdrawals allowed
    - A penalty equal to 10% of the withdrawal
    - A penalty equal to 20% of the withdrawal
    - Withdrawals disallowed altogether
  - Illiquid accounts earned a varying amount of interest
    - 21, 22, 23% interest varied randomly
  - Experiment removed the commitment accounts with 21% interest and 20% penalty and disallowed withdrawals
  - Subjects then had to allocate lots of money of \$50, 100 and 500 between the two accounts
    - Randomly chosen as to what they would receive
  - Rational consumer would choose the account the highest rate of interest

- Would choose commitment account with the earliest possible withdrawal date
- Results of experiment 1
  - Half of initial balances are allocated to the commitment account when it has the same interest rate as the liquid account
    - One quarter of initial balances are allocated to the commitment account when it has a 1% lower interest rate than the liquid account
  - When the commitment account and the liquid account have the same interest rate, stricter commitment accounts are more attractive
  - When the interest rate on the commitment account is higher than the interest rate on the liquid account, the relationship between commitment account allocations and illiquidity disappears
    - Commitment accounts with a 23% interest rate attract approximately 60% of the endowment regardless of their early withdrawal policy
  - Increasing the penalty for withdrawing causes a higher percentage of initial funds to be allocated to the commitment account
  - Failed to find any statistical evidence that suggested that withdrawals varied across each of the accounts
- Experiment two
  - Liquid account paid 22% interest and allowed withdrawals
  - Illiquid accounts also only paid 22% and varied the extent to illiquidity
    - Liquid account, and an account that imposed a 10% penalty on withdrawals
    - Liquid account, and an account that prohibited withdrawals
    - Liquid account, 10% penalty account, prohibition on withdrawals altogether
    - Liquid account, and safety valve account that prohibited withdrawals unless a financial emergency occurred
      - Not verified, so only imposed the psychological cost of lying
  - Participants told to allocate \$100 into each account, with a 50% probability they would receive this, or a 50% probability all would go into the liquid account
- Results of experiment 2
  - First combination
    - 10% penalty receives 46% of endowment
  - Second combination
    - No withdrawals receives 54% of endowment
  - Third combination
    - 34% no withdrawals, 16% 10% penalty
    - Total allocations to commitment accounts are not higher when two commitment accounts are available rather than one
  - Fourth allocation
    - Presence of safety valve account not statistically significant
  - Withdrawals
    - The balance ratios (ratio between final and initial endowment) for those in the safety valve condition do not differ when participants receive all of their endowment in a liquid account
      - Substantially lower in the 10% penalty and no early withdrawal conditions

## Exponential Discounting



## Exponential Discounting With Indifference



Consumer with will be indifferent between  $a$  and  $b$

## Time Consistency

Consider two payoff streams  $x$  &  $y$  that are identical except at  $t=1$  and  $t=2$  and DM is indifferent between them at  $t=0$

$$D(1)u(x_1) + D(2)u(x_2) = D(1)u(y_1) + D(2)u(y_2)$$

$$D(2) = D(1) \frac{u(y_1) - u(x_1)}{u(y_2) - u(x_2)}$$

Then at time 1:

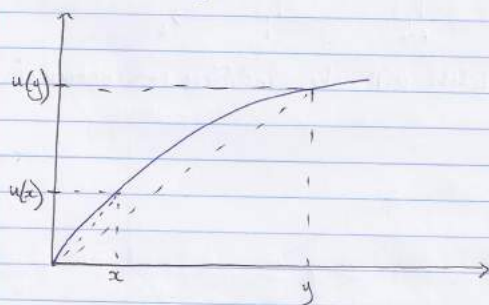
$$u(x_1) + D(1)u(x_2) = u(y_1) + D(1)u(y_2)$$

$$D(1) = \frac{u(y_1) - u(x_1)}{u(x_2) - u(y_2)}$$

$$\therefore D(2) = D(1)^2$$

$\therefore$  Discount function must be exponential

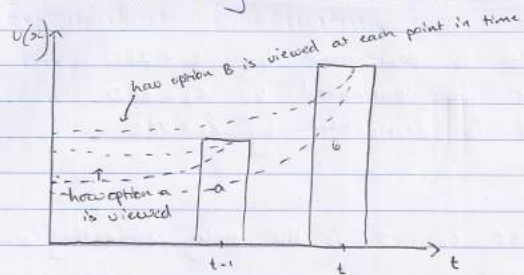
## Underestimating $\delta$



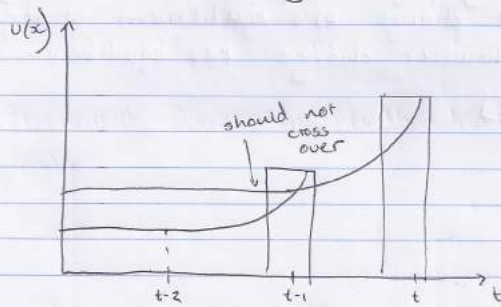
$$\frac{u(x)}{x} > \frac{u(y)}{y} \rightarrow \frac{u(x)}{u(y)} > \frac{x}{y}$$

Hence if  $\delta = \left(\frac{x}{y}\right)^{1/n}$ , this will underestimate  $\delta$

## Time Inconsistency and the Discount Function



Time consistency



Time inconsistency